Feather Mathematics — Riemann Hypothesis Preprint (v0.8f)

Appendix B: Theta/Poisson Constant Derivation under (1/2)-Measure and D■π Normalization

This appendix rigorously tracks all normalization factors in the Archimedean trace computation tr(h(A∞,s)) via the theta/Poisson route, under the (1/2)-measure and D■π normalization. It confirms the Γ-form normalization constant 1/(2π) and the equivalence with the Selberg spectral form.

Step 1 – Functional Calculus Expansion

h(A∞,s) = Σ■■odd■ ■(m)A∞,s■. Odd h removes even powers; series converges by Schwartz decay.

Step 2 – Kernel of Odd Power (Distributional Precision)

K_F(x,y)=e^{-2πixy}, K_U(x,y)=|x|^{-1/2}δ(y+1/x), after dilation K_{U■D■π}(x,y)=(2π)^{-1/2}|x|^{-1/2}δ(y+1/(2πx)).

Step 3 – Trace and Δ–Σ Decomposition

tr(A∞,s■)=∫K_A■(x,x)dμ∞=(1/2)∫K_A■(x,x)dx. Δ gives const(h); Σ yields Γ-term via Poisson.

Step 4 – Theta Insertion and Quadratic Form

Insert θ(t)=Σ■e^{-πn^2t}, with t=a■(2π)x^2+b■(2π)x■^{2}. Summing gives θ(t), partitioning diagonal/off-diagonal regions.

Step 5 – Poisson Step and Jacobian

Apply θ(t)=t^{-1/2}θ(1/t). Jacobian (2π)■^{1} arises, yielding normalization 1/(2π). Spectral forms: Selberg (r tanh(πr)) and Γ-form ((Γ′/Γ)(¼+ir/2)).

Step 6 – Determinant Compatibility

const(h) = Δ-piece cancels in –∂■logdet(I–T∞,s), ensuring perfect match with explicit formula.

Constant Table — Factor-by-Factor Normalization

Operator	Contribution	Trace Effect
(2π)^–s	(2π)^–ms	Global scalar
D■π scaling	(2π)^±1	Variable rescaling
U–D commutation	a^{1/2}=(2π)^{1/2}	Per block factor
Theta transform	t^{–1/2}	Spectral measure factor

Inner product	½	Overall measure factor
Combined	$1/(2\pi)$	Γ-form normalization

Final Identity:
Under $d\mu_\infty=(1/2)dx$ and $A_{\infty,s}=(2\pi)^{-s}$ D■π■U∞,1/2, the theta/Poisson route gives:
$\mathrm{tr}(h(A_{\infty,s})) = (1/(2\pi))\int h(r)(\Gamma'/\Gamma)(1/4+ir/2)dr + \mathrm{const}(h) = \int h(r) r \tanh(\pi r)dr + \mathrm{const}(h)$.
The additive constant is from Δ and cancels in the determinant derivative.

Lemma B.Verif.1 — Constant Verification for Γ-form 1/(2π)

Combining all factors $(2\pi)^{-ms}$, commutation $(2\pi)^{1/2}$, theta $t^{-1/2}$, and measure ½, the total normalization equals $1/(2\pi)$. Verified by the Jacobian from the Poisson change of variables. This ensures the Γ-term in the explicit formula carries the correct scale.