Constant Speed Fluid Flow Model of Electromagnetic Field: Alternative to Incompressible Flow Analogy

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Abstract

This paper revisits the fluid analogy for electromagnetic field theory, introducing an alternative model based on a constant fluid velocity and an electric analogy with the fluid density momentum. Unlike Maxwell's divergence of velocity fluid analogy, our approach relies on constant velocity (serving as an analogy to the speed of field) to drive the flow, providing a more direct analogy to field interactions and propagation. We develop this framework in detail, exploring the dynamics of fluid flow in a void, the oscillatory flow of the fluid from a perpendicular oscillatory motion of the source, and the effects of relative motion between source and observer. Our analysis yields a novel perspective on electromagnetic field theory, offering insights into fluid as a continuum and the connections between field theory and fluid dynamics. This work has implications for both physics and engineering and provides a foundation for potential applications in physics and beyond.

Keywords: Classical electromagnetism, continuity equation, electromagnetic field, electromagnetic wave, fluid dynamics, mathematical method Maxwell's equations, and vector calculus.

1 Introduction

The concept of modeling physical phenomena using fluid dynamics has a long and distinguished history, particularly in the context of electromagnetic theory. One of the most influential and early uses of this analogy was introduced by James Clerk Maxwell in the 19th century, who likened the behavior of the electromagnetic field to the flow of an incompressible fluid [1]. This flow model was used to visualize the abstract force field originally constructed by Michael Faraday, providing a more intuitive understanding of electric and magnetic interactions [2]. Maxwell assumed that the velocity of an incompressible fluid could represent the electric field. With this approach, Maxwell developed a physical picture of the electric and magnetic fields as interlinked components of a dynamic, flowing medium. The analogy served as an early heuristic tool, aiding in the formulation of the classical electromagnetic wave equations, which described the behavior of light as a transverse wave [3].

However, Maxwell's fluid analogy, while insightful, left several important aspects of electromagnetic field theory unexplored. In particular, it did not adequately address the propagation of electromagnetic waves or the transformation of light waves under different reference frames. The latter issue, concerning the invariance of light speed across reference frames, has been the subject of considerable debate and controversy, from early analyses of wave speed by

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Voigt and Poincaré to Einstein's revolutionary work on special relativity [4] [5] [6]. Although insightful, the incompressible fluid analogy did not capture the full complexity of the electromagnetic field and was eventually replaced by more sophisticated mathematical treatments, such as the introduction of vector and scalar potentials [7] and the development of relativistic electrodynamics [6]. However, these advancements came at the cost of losing the intuitive visualization provided by the fluid analogy.

In contrast to Maxwell's treatment, the present paper seeks to revisit and expand upon the fluid analogy by proposing an alternative model based on the divergence of the fluid mass. Unlike Maxwell's incompressible fluid, the flow considered here is driven by a constant velocity, where the fluid mass is involved in the flow rather than the transfer of momentum exhibited in an incompressible flow. In incompressible flow, the density is constant and the source of the divergence is the source of disturbance within the fluid, whereas in contrast, this paper analyzes a flow of constant velocity with the source serving as the source of the fluid mass into the system with no resistance to the flow. In this paper, I shall demonstrate that this model provides a direct analogy to the propagation of electromagnetic field.

Beyond offering an alternative analogy for classical electromagnetic theory, this paper has broader implications for both physics and engineering. It also provides a foundation for potential applications in computational physics, where fluid dynamics simulations could be used to model field behavior in complex systems. Furthermore, by examining the transformation of flow dynamics under relative motion, this paper connects the fluid analogy to foundational concepts in modern physics, such as frame invariance and electromagnetic wave propagation.

The goal of this paper is to explore the fundamental principles behind this alternative interpretation of electromagnetic phenomena, using fluid dynamics as a metaphor. The following sections will develop this framework in detail. Section 2 provides a historical and analytical background of the fluid analogy to electromagnetism, with a focus on Maxwell's incompressible fluid model. Section 3 will analyze the dynamics of fluid flow, considering conservative systems. It describes the condition of the fluid having a constant velocity and examines different source configurations, including linear and planar distributions. Section 4 will delve into the consequences of nonconservative flow and also demonstrate the generation of a perpendicular wavelike flow pattern and the effects of relative motion between source and observer. Section 5 applies the theory of compressible fluid flow to the electromagnetic field, by direct comparison, whereas Section 6 analyses its implications on the relativistic effect, in this section, the Lorentz force equation and the Lorentz factor are derived using the fluid analogy. Section 7 analyses analogical relationship of the model to the classical description of light and electromagnetic waves.

Through this detailed analysis, I aim to offer a novel perspective on electromagnetic phenomena, grounded in the principles of fluid dynamics, and to re-examine the connections between field theory and it fluid analogy. The insights derived from this model not only illuminate aspects of electromagnetic theory but may also inspire new approaches to understanding field phenomena in other areas of physics.

2 Background on Fluid Dynamics Analogy of Electromagnetic Field

The fluid analogy has played a significant role in the development of electromagnetic theory, particularly in the conceptualization of how fields behave and interact. This analogy, which draws parallels between the motion of fluids and the behavior of electromagnetic fields, can be traced back to the early works of notable scientists. One of the earliest mentions of a fluid-like substance in the context of magnetic phenomena came from William Gilbert, who, in his seminal work De Magnete proposed that the Earth's magnetic field could be understood as a result of a fluid-like substance flowing through the Earth [8]. Gilbert's model of magnetism was one of the first to suggest that natural phenomena could be explained using concepts of fluid dynamics.

In the 18th century, Benjamin Franklin and Alessandro Volta expanded on this idea by conceptualizing electricity as a form of fluid that could flow through conductors. Franklin likened electric charge to a fluid, calling it "electric fire," and believed that positive and negative charges were merely different amounts of the same fluid [2] [9]. Volta's work on the electrical battery further advanced the idea of electricity as a substance that could flow and accumulate, making it a central part of early electrical theory.

However, it was in the 19th century, with the work of Michael Faraday and James Clerk Maxwell, that the fluid analogy truly gained prominence in the context of electromagnetism. Faraday introduced the concept of "lines of force," which he visualized as fluid-like tubes extending from positive charges and terminating on negative charges [2]. He used this visualization to describe both electric and magnetic fields, emphasizing their dynamic and interconnected nature . These "lines of force" served as an early depiction of what would later become field lines in the modern understanding of electromagnetism.

Maxwell, building on Faraday's ideas, developed a comprehensive theory of electromagnetism that described how electric and magnetic fields interacted and propagated through space. In Maxwell's framework, the electromagnetic field was treated as a dynamic, fluid-like substance, continuously evolving and propagating through space. His set of equations, now known as Maxwell's equations, laid the foundation for classical electromagnetism and are often expressed in terms of a continuous, mathematical field framework rather than a tangible, fluid-like substance [1]. Despite this, Maxwell's use of a fluid analogy helped provide intuition for the underlying behavior of the fields, offering a bridge between classical mechanics and electromagnetism.

Maxwell's work was not just theoretical; he also employed the analogy of fluid dynamics to describe various aspects of electromagnetic behavior. For instance, he likened the electric field to the velocity field of a fluid, with the electric field lines representing the "flow" of this hypothetical fluid. Similarly, the magnetic field was compared to the vorticity or rotational motion of a fluid, with the vorticity representing the twisting of the fluid's flow. The pressure within the fluid analogy was used to describe the energy density of the electromagnetic field, while the incompressibility of the fluid reflected the idea that the electromagnetic field fills all of space and cannot be "compressed" in the same way as a traditional substance [10].

One of the more important aspects of Maxwell's use of the fluid analogy was his application of the incompressible fluid model, in which the field is assumed to have a constant "density" and undergoes steady, time-dependent motion. By treating the electromagnetic field as an incompressible fluid, Maxwell derived the equations governing electromagnetic waves, which are transverse waves that oscillate perpendicular to one another and to the direction of wave propagation. This was a significant insight, as it showed that electromagnetic waves, such as light, could propagate through space as a form of transverse motion, akin to waves on the surface of a fluid [3].

Despite the utility of the fluid analogy, Maxwell's incompressible fluid model had limitations. While it was successful in illustrating the wave nature of light, it did not fully unify the electric and magnetic fields under a single, coherent fluid framework. The analogy, although helpful for visualizing field interactions, could not account for the deeper mathematical relationships that bind the electric and magnetic fields together in a way that physicist believe them to be related. Specifically, Maxwell's fluid analogy could not fully reconcile the behavior of the electric and magnetic fields in dynamic systems, particularly in the context of the creation of electromagnetic waves.

As electromagnetism evolved into its modern form, the fluid analogy was gradually supplanted by more abstract and general mathematical formulations. The advent of vector calculus and the development of Maxwell's equations in their modern form provided a more rigorous, precise framework for describing electromagnetic phenomena [7]. However, the fluid analogy remains a powerful and intuitive tool for understanding the behavior of electric and magnetic fields, particularly in educational contexts and for conceptualizing complex field interactions.

In summary, the fluid analogy has played a pivotal role in the historical development of electromagnetic theory, providing early insights into the behavior of electric and magnetic fields and inspiring key concepts that would later be formalized in Maxwell's equations. While the fluid analogy has been superseded by more precise mathematical descriptions of electromagnetism, it continues to serve as a valuable pedagogical tool, offering intuitive explanations for the dynamic and interconnected nature of fields. The historical development of the fluid analogy highlights the creative and evolving nature of scientific thought, where concepts from one domain—such as fluid dynamics—can illuminate and enhance our understanding of seemingly unrelated phenomena, such as electromagnetism.

3 Theory of Conservative Fluid Flow

We shall consider a hypothetical substance that exhibits only two properties of ordinary fluids: freedom of motion and compressibility. This substance is not a real fluid, nor is it a hypothetical fluid used to explain actual phenomena. Instead, it's a mathematical construct which we shall use to analogize the electromagnetic field. For the purpose of this model, the "fluid" refers to a purely imaginary, continuous substance with density and constant velocity. We shall represent this fluid mathematically using the product of its density ρ_f and constant velocity $\mathbf{v}$, which is the density momentum $\mathbf{D}$.

For the system, we shall assume a hypothetical boundless three-dimensional system which is free from rigid boundaries and devoid of rigid bodies or matter and energy that could alter the fluid's motion through interactions. Instead of rigid bodies, the system only contains spherical sources and sinks which conveys the fluid into and out of the system. This assumption is necessary to create a steady and uniform flow unaffected by external influences. For the fluid in this hypothetical system, there are no vibrational properties (since there is no rigid body). The fluid enters and exits the system through the sources and sinks, and when within the system, because the system is devoid of matter, the fluid maintain a constant velocity $\mathbf{v}$ relative to the source as it diverge into the void of the system. This fluid system can be visualized as a vacuum space (devoid of matter, energy and potential), where fluid begins to sip out from a concavity within the space and diverge outward with a constant speed. This is physically imposible, but only served as a visualization for the fluid system which we shall analyze.

The conservation of mass for the fluid with density ρ_f and velocity field $\mathbf{v}$ is expressed by

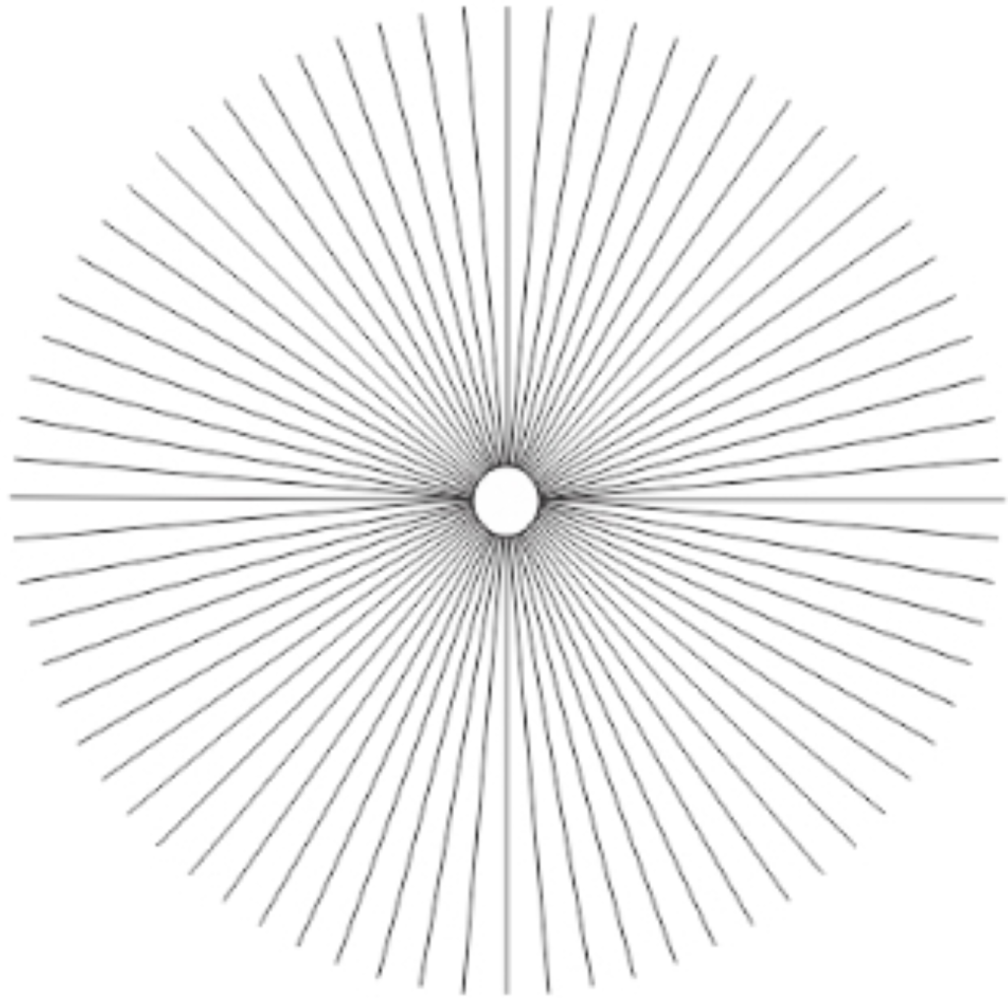

Figure 1: *The divergence of the fluid from the source is represented by the streamline. The fluid diverge outward at constant speed, whereas the density spread out. Each streamline signifies a constant density (as represented by the equation* $\nabla\rho = 0$*) which spread out as it diverge from the source.*

the continuity equation [11]

$$\frac{\partial \rho_f}{\partial t} + \nabla \cdot (\rho_f \mathbf{v}) = \mathcal{R}, \tag{1}$$

where $\mathcal{R}$ is the volumetric source strength. For the system, the density ρ_f decreases as the fluid diverges from the source, expanding into the void, whereas, the velocity $\mathbf{v}$ remains constant due to momentum conservation. The constant velocity signifies that $\nabla \cdot \mathbf{v} = 0$, thereby demonstrating a uniform and conservative flow.

As the fluid diverges outward, a given mass of the fluid occupies more volume, that is the density decreases outward with the flow. However, at every infinitesimal volume, the quantity of fluid that flow through the volume is constant as long for the conservative condition of a relative stationary from the chosen volume. This is the condition of uniform and steady flow, and the condition of conservative density momentum within a source-free control volume (that is, when $\mathcal{R} = 0$). Though the density display spatial variation as it diverges, it is time-independent at every point of space for the relative stationary source. Therefore, in the continuity equation of (1), $\frac{\partial \rho_f}{\partial t} = 0$, so that the continuity equation reduces to

$$\nabla \cdot \mathbf{D} = \mathcal{R}. \tag{2}$$

This equation describes the divergence of momentum density, where the sources within the control volume contribute to $\mathcal{R}$. However, when there is no source within the control volume, $\mathcal{R} = 0$.

Since the fluid only exhibit divergence and not irrotational flows, the momentum density can also be expressed as the gradient of a scalar potential ψ (that is, $\mathbf{D} = -\nabla\psi$ [12]) which can be substituted into the steady-state continuity equation of (2) to gives the Poisson's equation of the flow [13]

$$\nabla^2 \psi = -\mathcal{R}. \tag{3}$$

In source-free regions where $\mathcal{R} = 0$, this equation also simplifies to the Laplace's equation [13]

$$\nabla^2 \psi = 0. \tag{4}$$

The potential function ψ represents the scalar field governing the fluid's behavior. However, in this paper, the density momentum of the flow shall be the main form which would be used in expressing the flow system, rather than the scalar or vector potential.

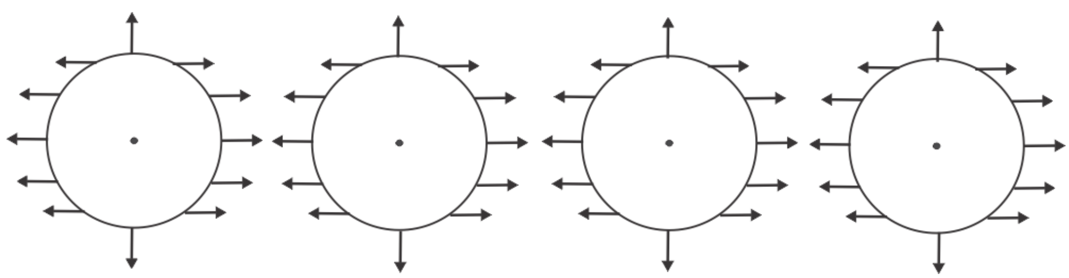

Figure 2: *The opposite flowing fluid of the axis of length of the linear source, cancels one another out. The only non-zero divergence of the fluid is in the perpendiular axes, which combines the density momentum of the other axes together, in order to conserve momentum.*

3.1 Symmetrical Configurations of Sources

1. **Point Source** For a point source symmetrically injecting fluid into the system, (2) applies. The divergence theorem can be applied over the spherical control volume to get the integral equation, given as [14]

$$\int_V \nabla \cdot \mathbf{D}\, dV = \int_{\partial V} \mathbf{D} \cdot d\mathbf{A} = S, \tag{5}$$

where $S = \int \mathcal{R}\ dV$ is the strength of the source. The magnitude $|D|$ at a distance r from the source is

$$|D| = \frac{S}{4\pi r^2}. \tag{6}$$

This describes the inverse-square law dependence of momentum density on distance from a spherical source.

2. **Linear Source** A linear source can be approximated as an array of point sources, assuming cylindrical symmetry. This approximation shall be assumed in future sections. The momentum density has no component along the axis of the cylinder due to symmetry, leaving only radial components(Fig. 2, Fig. 3). Mathematically, using the flux form of Green's theorem to integrate the area of divergence of the fluid, the equation is given as [15]
$$\int_A \nabla \cdot \mathbf{D}\, d\mathbf{A} = \int_{\partial A} \mathbf{D} \cdot dl = \boldsymbol{\lambda}, \tag{7}$$
where $\boldsymbol{\lambda}$ is the linear source strength and7 defined as $\boldsymbol{\lambda} = \int \mathcal{R}\, d\mathbf{A}$. The magnitude of $|D|$ at a distance r from the axis is
$$|D| = \frac{\lambda}{2\pi r}. \tag{8}$$
This shows an inverse linear dependence on distance, which is true for a linear source of field.

3. **Planar Source** A plane source, modeled as a collection of point sources, exhibits symmetry such that momentum density exists only perpendicular to the plane. We can define the density momentum magnitude as [16]
$$|D| = \frac{\sigma}{2}, \tag{9}$$
where σ is the planar source strength and defined as $\sigma = \int \mathcal{R}\, dl$. Here, D is uniform, independent of distance from the plane.

It is important to note mathematically and with the aid of the visual model that the linear and planar configurations must be infinite for the equations of (8) and (9) to be true. Onwards, an infinite linear and planar configurations shall be assumed where they apply.

From this analyses we can conclude that the momentum density $\mathbf{D}$ is dimensional dependence—it varies with the spatial configuration of the source (point, line, plane). The equations elegantly connect density, flow, and symmetry, with Poisson's and Laplace's equations central to the analysis. This framework provides foundational insights into conservative flows in idealized systems and could inform models of real-world scenarios under constrained conditions.

3.2 Energy Density of the Fluid Flow

The energy density of the fluid is a crucial concept in understanding the behavior of fluids in motion. For a stationary source, the energy density of the fluid can be derived from the density momentum $\mathbf{D}$. The energy density U of the fluid is defined as the energy per unit volume of the fluid. For a stationary source, the energy density can be expressed as
$$U = \rho_f\, v^2, \tag{10}$$
where ρ_f is the fluid density and v is the velocity of the fluid. Using the definition of the density momentum $\mathbf{D} = \rho_f\, \mathbf{v}$, the energy density can be rewritten as
$$U = \mathbf{D} \cdot \mathbf{v}. \tag{11}$$

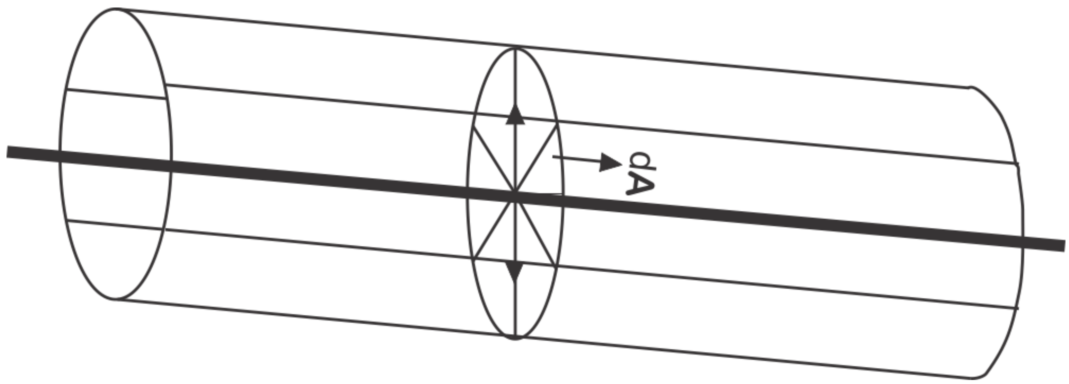

Figure 3: *The fluid from an infinitesimal part of the linear configuration spread over the area and not the volume as it does in the case of an isolated spherical charge. The normal of the area $\hat{k}$ is perpendicular to the area and is the direction of current.*

From (10), it can be observed that the energy density of the fluid is equivalent to the dynamic pressure. Therefore, the energy density can be expressed as the force exerted by the fluid on an imaginary surface element dA on the source. The force exerted by the fluid on the surface element is given by

$$dF = U\ dA. \tag{12}$$

The dynamics of fluid flow can be effectively described in terms of energy density or pressure. Crucially, the force exerted by the fluid as it flows into a nearby sink is precisely the force experienced by the sink, underscoring the importance of considering fluid flow in the context of its interactions with surrounding objects.

4 Theory of Non-Conservative Flow from a Linear Source

Unlike conservative flow, non-conservative flow exhibits spatially and temporally varying densities due to source motion or continuous density. This process produce a rate of change in density momentum relative to a control volume in the vicinity. In this section, the non-conservative flow from a linear source shall be examined under specific conditions. The flow field of a non-conservative linear symmetry configuration is analyzed, establishing relationships between the linear source and the generated field. The symmetry simplifies the analysis by constraining flow and current properties. For a stationary changing source strength or a moving linear source, point sources within the linear configuration are assumed to have a dynamic source strength or motion along its length relative to a point in the vicinity, respectively. In the moving point source symmetry, source motion affects fluid flow, introducing a component of flow in the direction of the sources motion (Fig. 4). The density momentum then has two components: longitudinal ($\mathbf{D}_L$), as stated in (1) as $\mathbf{D} = \rho_f\ \mathbf{v}$, and transverse ($\mathbf{D}_T$), which is given as

$$\mathbf{D}_T = \rho_f\ \mathbf{u}, \tag{13}$$

where $\mathbf{u}$ is the speed of the flow in the component as an effect of the motion by the source, which have the same speed.

Figure 4: *For every outflowing fluid from the linear configuration, there is a perpendicular velocity* **u** *equal to and produced by the motion of the current. This gives the flow two components: The default longitudinal component and the current's motion-induced transverse component.*

4.1 Rate of Change of Source Strength

The temporal evolution of the source strength is characterized by the current $\dot{S}$, which encapsulates variations in source intensity over time. For a stationary source, a nonzero $\dot{S}$ emerges solely due to intrinsic fluctuations in source strength. From (5), this current can be expressed in terms of the density momentum $\mathbf{D}$ as

$$\frac{d}{dt}\oint_{\partial V} \mathbf{D} \cdot d\mathbf{A} = \dot{S}, \tag{14}$$

where the integral is taken over a closed surface ∂V enclosing the source. Differentiating with respect to the surface element leads to the local differential form

$$\frac{\partial \mathbf{D}}{\partial t} = \dot{\boldsymbol{\sigma}}, \tag{15}$$

where $\dot{\boldsymbol{\sigma}}$ represents the instantaneous rate of change of the area current density.

For a stationary, linear source with time-dependent source strength, the density momentum $\mathbf{D}$ can be related to the linear source strength $\boldsymbol{\lambda}$, yielding

$$\dot{S} = \frac{d}{dt}\left(\boldsymbol{\lambda}\int \frac{1}{dl}\, d\mathbf{A}\right), \tag{16}$$

given that $\mathbf{D} = \frac{\boldsymbol{\lambda}}{dl}$, which follows from (7). Here, dl represents an infinitesimal segment along the source, and the integral term quantifies the contribution of the source's linear structure within the control volume. By recognizing that the integral corresponds to the divergence length in the chosen coordinate system, we obtain

$$\dot{S} = \frac{d}{dt}(\boldsymbol{\lambda}\mathbf{r}). \tag{17}$$

Applying the product rule of differentiation,

$$\dot{S} = \boldsymbol{\lambda}\frac{d}{dt}\mathbf{r} + \mathbf{r}\frac{d}{dt}\boldsymbol{\lambda}, \tag{18}$$

where $\frac{d}{dt}\mathbf{r} = \mathbf{v}$ represents the velocity of field propagation, while $\dot{\boldsymbol{\lambda}}$ denotes the rate of change of the linear source strength. Consequently, the governing equation simplifies to

$$\dot{S} = \boldsymbol{\lambda}\mathbf{v} + \dot{\boldsymbol{\lambda}}\mathbf{r}. \tag{19}$$

For a stationary linear source with time-dependent strength, the term $\dot{\boldsymbol{\lambda}}$ characterizes variations in source intensity, whereas $\boldsymbol{\lambda}\mathbf{v}$ remains a constant contribution. Since $\mathbf{r}$ represents a characteristic length scale relative to the system's symmetry, the velocity $\mathbf{v}$ corresponds to the field's transport speed in the given configuration. It is essential to emphasize that the current formulation is inherently symmetry-dependent but remains independent of any external test source. This follows from the derivation, which relies on a prescribed control volume dV rather than any particular test configuration.

4.2 Fields of Non-Conservative Flow Induced by Point Source Currents in Linear Symmetry

Consider a linear source composed of point sources moving as a current within a system exhibiting linear symmetry. The resulting field distribution depends fundamentally on the relative motion of these sources. By substituting the definition of the density momentum, given by $\mathbf{D} = \rho_f \mathbf{v}$, into (7), we obtain

$$\mathbf{v}\int \rho_f \, dl = \lambda, \tag{20}$$

where λ represents the linear source strength.

For a steady-state current of point sources confined within the symmetry, the rate of change of the linear source strength is zero, assuming that the flow remains conservative and sources are not displaced away from the axis of motion. Substituting $\lambda = \frac{\dot{S}}{v}$ into (20), we arrive at

$$\int \rho_f \, dl = \frac{\dot{S}}{v^2}. \tag{21}$$

The infinitesimal path element dl can be expressed in vector form as $\hat{j} \cdot d\mathbf{l}$, with $\hat{j}$ denoting the unit tangent vector along the loop. For a moving current within the linear source, the current is proportional to the speed of the source current rather than the divergence velocity, a property previously established for stationary dynamic source strengths. Following a similar derivation, one finds that in the absence of a non-conservative flow (displacement current), the current of the point sources satisfies

$$\dot{S} = \boldsymbol{\lambda}\mathbf{v}. \tag{22}$$

To incorporate the effects of motion-induced current, the equation is modified by introducing the ratio of the current velocity $\mathbf{u}$ to the divergence velocity $\mathbf{v}$, yielding

$$\int \rho_f \frac{\mathbf{u}}{\mathbf{v}}(\hat{j} \cdot d\mathbf{l}) = \frac{1}{v^2}\left(\dot{S}\frac{\mathbf{u}}{\mathbf{v}}\right). \tag{23}$$

To refine the left-hand side, we introduce the unit vector, defined as

$$\hat{\rho} = \frac{u}{v}\hat{j}, \tag{24}$$

where the ratio $\frac{u}{v}$ serves as a proportionality factor between the two unit vectors. Consequently, the density scalar ρ_f transforms into the density vector $\boldsymbol{\rho}_f$, oriented in the direction of $\hat{j}$, leading to

$$\int \boldsymbol{\rho}_f \cdot d\mathbf{l} = \frac{1}{v^2}\lambda\mathbf{u}, \tag{25}$$

where $\boldsymbol{\lambda}\mathbf{u}$ represents the current associated with point sources in the linear symmetry. It signifies a moving form of source current and is different from the stationary dynamic source strength current of (22). Employing Stokes' theorem, the equation is transformed from the integral to the differential form, yielding

$$\nabla \times \boldsymbol{\rho}_f = \frac{1}{v^2}\frac{\partial \mathbf{D}}{\partial t}. \tag{26}$$

This equation reveals the intrinsic rotational characteristics of the density vector field $\boldsymbol{\rho}_f$ within linear symmetry configurations. The emergence of $\dot{S}$ is fundamentally linked to the rotational properties of the field, illustrating that the density vector field exists as a direct consequence of motional currents. The tangential component of the field, encoded in $\boldsymbol{\rho}_f$, encapsulates the system's response to a linear charge density $\boldsymbol{\lambda}$.

To incorporate displacement current effects—arising from temporal variations in the linear source strength or non-conservative flow of the source current within the symmetry— (25) generalizes to

$$\int \boldsymbol{\rho}_f \cdot d\mathbf{l} = \frac{1}{v^2}\left(\lambda\mathbf{u} + \dot{\boldsymbol{\lambda}}\mathbf{r}\frac{u}{v}\right), \tag{27}$$

where the term $\dot{\boldsymbol{\lambda}}\mathbf{r}\frac{u}{v}$ accounts for displacement current contributions. In a system exhibiting linear symmetry, the current generated by point sources is directly proportional to the linear flow density and velocity. The coupling between current, density momentum, and the density vector field is fully captured through integral and differential formulations, highlighting the rotational structure inherent in such systems.

For non-conservative flow, the general form of the continuity equation remains

$$\nabla \cdot \mathbf{D} = -\frac{\partial \rho_f}{\partial t}, \tag{1}$$

where the temporal evolution of the density field ($\frac{\partial \rho_f}{\partial t} \neq 0$) induces corresponding variations in the momentum density. To express this relation in terms of the source current within the system's configuration, (1) is multiplied by the unit vector $\hat{\rho}$. Given that $\mathbf{D} = \rho_f\mathbf{v}$, we obtain

$$(\nabla \cdot \rho_f\mathbf{v})\frac{u}{v}\hat{j} = -\frac{\partial \rho_f}{\partial t}\hat{\rho}. \tag{28}$$

Rearranging the left-hand side,

$$(\nabla \cdot \rho_f\mathbf{u})\hat{j} = -\frac{\partial \rho_f}{\partial t}\hat{\rho}. \tag{29}$$

By recognizing that the density momentum derived here, corresponds to the transverse component of the field as given in (10), and the left-hand term is identified as the curl of the transverse field component $\mathbf{D}_T$. Defining the density vector $\boldsymbol{\rho}_f$ as

$$\boldsymbol{\rho}_f = \rho_f \hat{\rho}, \tag{30}$$

the governing equation simplifies to

$$\nabla \times \mathbf{D}_T = -\frac{\partial \boldsymbol{\rho}_f}{\partial t}. \tag{31}$$

This result explicitly confirms the rotational characteristics of the density momentum field in non-conservative systems. The presence of $\nabla \times \mathbf{D}_T$ signifies the emergence of rotational components in the density momentum field due to temporal variations in the density distribution. Consequently, the system develops a non-conservative structure, where ∇, $\mathbf{D}_T$, and $\boldsymbol{\rho}_f$ form a mutually orthogonal triad.

It is crucial to note that the formulation here pertains exclusively to the transverse component of the density momentum field, omitting any longitudinal contributions. The absence of the longitudinal term underscores that the density vector $\boldsymbol{\rho}_f$ arises strictly from transverse density momentum dynamics and remains independent of any parallel (divergent) flow component.

4.3 Field Relativity and Transformation

The density momentum analysed so far is in the reference frame of a static position in the system. By the principle of relativity, the motion of the position or control volume chosen relative to the flow source also affects the density momentum observed by the position. Therefore, the flow velocity $\mathbf{v}$ is the flow speed relative to the observer in the given position. In this sub-section, the relativistic transformation of density momentum in the context of fluid dynamics shall be explored, particularly when the motion of the observer influences the density momentum field. The previous analysis of the static observer is expanded to account for the effects of relative motion. This transformation is crucial for understanding how moving observer influence the dynamics of the density momentum in a fluid system. The primary goal is to establish the theoretical framework that governs how a static density momentum field evolves when the sources or observers are in motion relative to one another.

For a stationary fluid source, the density momentum $\mathbf{D}$ is governed by a divergence-free condition of (2). The equation reflects a conservative field where the density remains constant and there is no net flux of momentum through the system. Essentially, this describes a static configuration in which the density momentum is balanced and stable. In such static system, no net change occurs over time, and hence the density momentum field is static. This setup serves as a base model for understanding how the dynamics of fluid density momentum evolve when the system is subject to motion.

Consider a scenario in which the test source is in motion relative to the field (which shall henceforth be used to express the observer). The motion of the test source is observed as a relative motion of the field by the test source. Although, the source is stationary relative to the system, the velocity $\mathbf{v}$ of the density momentum relative to the observer is no longer constant, but instead depends on the relative velocity $\mathbf{u}$ between the main source and the test source. Therefore, relative to the test source, the conservative field relative to the system, is non-conservative. Therefore, in the reference frame of the test source the field is dynamic, though it is actually stationary relative to the system.

Let us denote the relative velocity of the density momentum in the reference frame of the test source as $\mathbf{v}'$, where $\mathbf{v}' = \mathbf{v} + \mathbf{u}$. Here, $\mathbf{v}$ is the velocity of the density momentum in the reference frame of the system (or in the rest frame of the test source), and $\mathbf{u}$ is the relative velocity between the source and the test source. Substituting the modified velocity into the equation for the density momentum, we obtain

$$\nabla \cdot \mathbf{D}' = \nabla \cdot \rho_f(\mathbf{v} + \mathbf{u}) = 0, \tag{32}$$

where $\mathbf{D}'$ represents the relative density momentum as observed by the test source. By applying the vector product rule for divergence of a product of functions, we can expand this equation to become

$$\nabla \cdot (\rho_f \mathbf{v}) + \nabla \cdot (\rho_f \mathbf{u}) = 0 \tag{33}$$

The term $\nabla \cdot (\rho_f \mathbf{v})$ corresponds to the rate of change of density momentum in the moving frame, while the term $\nabla \cdot (\rho_f \mathbf{u})$ represents the change in the density momentum due to the relative motion between the sources.

The relative motion between the sources introduces a time-varying term in the equation, indicating that the field is no longer conservative. Therefore, in the moving frame, $\nabla \cdot (\rho_f \mathbf{u})$ is the rate of change of the density $\frac{\partial \rho_f}{\partial t}$. This leads to the following expression for the rate of change of the density momentum, given as

$$\nabla \cdot \mathbf{D} = -\frac{\partial \rho_f}{\partial t} \tag{34}$$

This equation represents a non-conservative field. The presence of the time derivative indicates that the density momentum is no longer static, and its dynamics, in this case, depend on the relative motion between the sources. This non-conservativeness suggests that the relative motion produces a flux of momentum in the fluid, with the density changing over time due to the motion of the sources. Thus, a dynamic situation is created where the density momentum field is no longer in equilibrium, and the relative motion of the sources creates a time-varying effect in the field. This is analogous to a situation where a moving source introduces a non-conservative elements to the field, leading to a time-dependent evolution of the fluid dynamics. Therefore, the field of a moving source or the moving test source is always observed in the reference frame of the test source as a non-conservative field.

The non-conservative nature of the field becomes evident when the relative motion effect on the fluid dynamics is examined. In the static case, the density momentum field is conservative and governed by (2). However, in the moving frame, the relative velocity introduces a time-dependent term that alters the dynamics of the field. (34) represents the evolution of the density momentum field under the influence of relative motion. The time dependent term signifies the creation of a non-conservative field, where the rate of change of the fluid density momentum is directly related to the relative velocity of the sources. By the relativity principle, in the frame of a linear source moving in the axis of its length, a static linear has a transverse density momentum $\mathbf{D}'_T$, the same as (31).

The analysis of density momentum in the context of relative motion provides a new perspective on fluid dynamics. By considering the motion of the sources relative to the test source, we derive equations that govern the evolution of the density momentum field, which becomes time-varying and non-conservative. This transformation introduces rotational flow patterns and dynamic changes in the field, similar to how the flow in a moving fluid system is affected by the relative motion between sources.

The key conclusion is that motion introduces a time-dependent variation in the density momentum, leading to a non-conservative field. This theory provides a foundation for understanding how the relative motion of sources modifies the fluid flow, and establishes a mathematical framework for analyzing dynamic fluid systems with moving sources.

4.4 The Wave Equations of the Fields

The density momentum $\mathbf{D}_T$, which represents the momentum of the fluid due to the motion of the current, is observed to remain constant with constant current velocity and when the velocity changes (either magnitude, direction or both) the density momentum changes correspondingly. Similarly, the fluid density vector $\boldsymbol{\rho}$, which is perpendicular to both the current (or density momentum) and the direction of flow from the linear source, also changes when the current velocity changes. This behavior is indicative of the relationship between the current, fluid density, and the momentum carried by the fluid.

Consider a scenario where the current is not steady, but instead changes steadily or randomly by acceleration of the point sources. The changing velocity of the current induces a corresponding variation in the density momentum. To derive the mathematical description of this dynamic behavior of the current, we begin with the non-conservative equation in (1). Assuming a conservative control volume of observation, through which fluid from the source passes, we invoke the condition $\nabla \cdot \mathbf{D} = 0$ (which holds for the divergence in free space). Taking the curl of (31) and applying the vector calculus identity for the curl of a curl, then

$$\nabla(\nabla \cdot \mathbf{D}) - \nabla^2 \mathbf{D} = -\frac{\partial}{\partial t}(\nabla \times \boldsymbol{\rho}), \tag{35}$$

where $\nabla \cdot \mathbf{D} = 0$, reducing the first term to zero. Substituting (26) into the right-hand side, we simplify this expression and obtain the following wave equation for the density momentum $\mathbf{D}$

$$\frac{\partial^2 \mathbf{D}}{\partial t^2} = v^2 \nabla^2 \mathbf{D}. \tag{36}$$

The derived equation is the wave equation [19]. This demonstrates that the transverse density momentum motion can travel as a wave transfering it energy from it source, outward into the surrounding space, where v, which is the speed of the fluid, also represents the speed of the transverse wave generated by the accelerating current. It also demonstrates that a dynamic current can generates a wave, with the fluid speed v acting as the propagation speed of the wave. By fluid dynamics, the flow speed being equal to the wave speed demonstrates that the wave created is a fluid flow wave or an oscillatory component in the flow, which creates a wave profile. The non-conservative current equation of (27) is not used for the assumed conservative condition.

Similarly, the wave equation in terms of the fluid density vector $\boldsymbol{\rho}$ can be derived by taking the curl of the equation for the density vector in (26). Using the fluid conservation condition $\nabla \cdot \boldsymbol{\rho} = 0$, the resulting equation is simplified to

$$\nabla^2 \boldsymbol{\rho} = \frac{1}{v}^2 \frac{\partial}{\partial t}(\nabla \times \mathbf{D}). \tag{37}$$

Substituting the expression for $\nabla \times \mathbf{D}$ from (31) and rearranging, the following wave equation for the density vector $\boldsymbol{\rho}$, is obtained

$$\frac{\partial^2 \boldsymbol{\rho}}{\partial t^2} = v^2 \nabla^2 \boldsymbol{\rho}, \tag{38}$$

where v is the speed of wave. This equation describes the wave property of the fluid density vector.

These wave equations reveal that the fluid can create a wave profile and transport energy density in the transverse component rather than through longitudinal flow. The longitudinal component, on the other hand, can only transport energy density by longitudinal flow, whereas by the wave equation, the transverse flow can produce a wave characteristic. This implies that a sinusoidal wave can be generated by sinusoidally accelerating the current. While mere acceleration can create variations in density momentum, a full wave variation requires oscillatory motion of the current. This oscillatory motion produces a well-defined frequency and wavelength, dependent on the oscillation characteristics. Notably, the wave equation merely demonstrates the possibility of wave motion, without necessarily implying a sinusoidal waveform, which can only be achieved through the actual sinusoidal motion of the current.

5 Application to Electromagnetic Field

In the preceding sections, the flow of a compressible fluid originating from various types of sources has been fully analysed. I assume that the reader is familiar with the concept of the electromagnetic field and has likely recognized the analogies between the fluid flow field and the electromagnetic field. It is crucial to note that fields, including the electromagnetic field, are hypothetical constructs used to describe action-at-a-distance. They are not physical entities in themselves. The analogy presented here serves as a simplified model for understanding the electromagnetic field produced by an isolated charge or electric conductor under specific conditions. In such a scenario, the influence of the test charge used to determine the force and its direction—the field—can be considered negligible. Under these simplified conditions, the flow field can be analogically compared to the electromagnetic field.

5.1 Analogy to Electrostatic Field

In electrostatics, the behavior of the electric field $\mathbf{E}$ is determined by Gauss's law which is given as [20]

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \tag{39}$$

where ρ is the charge density, and ϵ_0 is the permittivity of free space. The field $\mathbf{E}$, can also be expressed in terms of the electric potential ϕ, as [21]

$$\mathbf{E} = -\nabla \phi. \tag{40}$$

This potential satisfies Poisson's equation, given as [22]

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}. \tag{41}$$

In regions where $\rho = 0$, the equation reduces to Laplace's equation, given as [22]

$$\nabla^2 \phi = 0. \tag{42}$$

In the fluid model, the density momentum $\mathbf{D}$, follows similar governing equations. (2) mirrors Gauss's law of electric field, with $\mathbf{D}$ analogous to $\mathbf{E}$ and the source term $\mathcal{R}$ and ρ for the fluid and electric field respectively. The equations are structurally identical. The term $\mathbf{E}$ corresponds to $\mathbf{D}$, and ϵ_0 acts as a scaling factor in the electrostatic system that is implicitly unity in the fluid model. In integral form, by the divergence theorem, Gauss's law in integral form is [20]

$$\int_V \nabla \cdot \mathbf{E} \, dV = \int_{\partial V} \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}, \tag{43}$$

where $Q = \int_V \rho \, dV$ is the total charge within the volume. This equation is an analogy of the density momentum equation given in (5). The structure of the integral equations is identical. The flux of $\mathbf{E}$ through a surface corresponds to the flux of $\mathbf{D}$, with charge Q analogous to source strength S.

For a point charge Q, Gauss's law in spherical symmetry gives

$$E = \frac{Q}{4\pi\epsilon_0 r^2}. \tag{44}$$

This equation is similar to (6). Both fields exhibit an inverse-square dependence on the distance r, with Q and S acting as the source magnitudes.

For an infinite line charge with linear charge density λ, Gauss's law in cylindrical symmetry gives [23]

$$E = \frac{\lambda}{2\pi\epsilon_0 r}. \tag{45}$$

For an infinite linear source with strength λ, the density momentum is stated in (8). The fields exhibit an inverse relationship with r, governed by the linear source term λ.

For an infinite charged plane with surface charge density σ, the electric field is uniform, given as [24]

$$E = \frac{\sigma}{2\epsilon_0}. \tag{46}$$

For an infinite planar source with strength σ, the density momentum is also uniform, as derived in (9). Both fields are constant, independent of the distance from the plane, due to the symmetry of the source. Similarly, the potential functions ϕ for electric field as given in (41) and ψ as given in (3) satisfy identical Poisson equations, linking the source term to the spatial variation of the field.

The mathematical framework governing the density momentum in the fluid model aligns closely with the established equations of electrostatics. Both systems share: Divergence equations relating the field to a source term; Integral forms describing flux conservation; Inverse-square, inverse-linear, and constant field dependencies for point, linear, and planar sources, respectively; and Potential functions satisfying Poisson and Laplace equations. This structural similarity reinforces the analogy between fluid density momentum and electric fields, demonstrating the universality of mathematical principles across physical systems.

5.2 Field Density and Electrostatic Field

Applying the analogy of the fluid flow to the electric field, we can assume that the electrostatic field of a charge, which is the field that permanently exists with charge, emanates from the charge at a finite speed equal to the known speed of light. The assumption of finite speed agrees with modern theories of finite effect-speed relative to field interactions.

In the fluid flow theory, the density momentum is related to density by the constant speed of flow. By analogy, we can apply this concept to the electrostatic field, where the density momentum corresponds to the electric field and the speed of flow corresponds to the speed of light, which henceforth, shall be expressed as the speed of field. This leads to the introduction of a new property: The Volumetric Field Density, which is simply the ratio of the electric field to the speed of field in the corresponding component.

Analogically, this is expressed as

$$\mathbf{E}_s = \beta \mathbf{c}, \tag{47}$$

where $\mathbf{E}_s$ is the electrostatic field, β is the volumetric field density, and $\mathbf{c}$ is the speed of field. It is important to note that the Field Density is a scalar quantity, and should not be confused for the magnetic field, which is a vector quantity that only exists in electrodynamic conditions. Applying the fluid analogy, the longitudinal density momentum is an analogy to the electrostatic field.

By analogy with the fluid flow, the volumetric field density can be defined in terms of the linear field density, incorporating the surface area term. This yields an equation equivalent to Coulomb's Law:

$$\frac{\eta}{4\pi r} c = \frac{Q}{\epsilon_0 4\pi r}, \tag{48}$$

where η is the linear field density. Simplifying, we obtain

$$\frac{Q}{\epsilon_0} = \eta c. \tag{49}$$

This equation demonstrates the equivalence of electric flux to field density. In the fluid analogy, the linear fluid density remains constant as the fluid diverges outward in a streamline, implying that the flux is also constant. This concept is also valid in electrostatics, where we introduce the linear field density–which serves an analogy to the linear fluid density–as the ratio of electric flux to the speed of light. In subsequent sections the volumetric field density shall be of more use, and we shall simply call it The Field Density henceforth.

5.3 Application to Electrodynamics and Magnetic Field of a Linear Conductor

In fluid dynamics, the concept of density momentum in a non-conservative compressible flow provides an interesting analogy to the behavior of electric and magnetic fields in electrodynamics. By drawing parallels between the mathematical structures governing fluid flow and the equations of electromagnetism, we can gain deeper insights into how density momentum in fluids behaves similarly to electromagnetic field in electrodynamics.

In fluid dynamics, for a non-conservative compressible flow, the density is not constant at any given position. The continuity equation in this case retains its form as in (1). This equation reflects the changing density momentum emanating from a source, similar to the way in which electric field can evolve in the presence of a varying magnetic field and vice versa in electrodynamics. For an electric current moving within a linear conductor, by applying the flow analogy to the electromagnetic field, we shall assume that the field produced by the current has a transverse component parallel to the direction of current. This allows the electrodynamic field $\mathbf{E}_d$, to be an analogy to the transverse density momentum. The electrodynamic field is the field created by the motion of the charge and is perpendicular to the electrostatic field $\mathbf{E}_s$ which is the natural field of the charge. By applying the fluid analogy, the electrodynamic field is therefore defined as

$$\mathbf{E}_d = \beta \mathbf{v}, \tag{50}$$

where $\mathbf{v}$ represents the speed of the current. Taking the linear conductor to be made up of equal positive and negative charges that creates their positive and negative electrostatic field respectively, they can all cancel out one another to produce a neutral electrostatic field. Thereby, the only field left is the electrodynamic field of the current. This is true for a long thin wire, which does not have an electrostatic field, but only electrodynamic field produced by an electric current on the wire.

Using the flow model, the magnetic field can also be defined as an analogy to the density vector. Therefore, the magnetic field would be related to the field density by

$$\mathbf{B} = \beta \hat{b}, \tag{51}$$

where $\mathbf{B}$ represents the magnetic field vector and $\hat{b}$ represent the magnetic field unit vector, which would be analogically defined as

$$\hat{b} = \frac{v}{c}\hat{j}, \tag{52}$$

serving as an analogy to (24), where $\hat{j}$ is the unit vector of the rotational property of the magnetic field around the wire. Substituting this equation into (51), yields the scalar form

$$Bc = \beta v. \tag{53}$$

Substituting this equation into the electrodynamic equation of (50), the general relation of the electrodynamic and magnetic field is derived. Given as

$$E_d = Bc. \tag{54}$$

This equation demonstrates that the electrodynamic field intensity is equal to the product of the magnetic field intensity and the speed of the field. This is the electromagnetic field equation, verifying our flow model. However, from our model, it is important to note that the equation applies to every electrodynamic field and not limited to electromagnetic wave. It demonstrates that when there is an electrodynamic field, there is also a magnetic field, with their relationship being the speed of field.

The flow model, as analysed in Section 4 also account for the Maxwell's curl equations: Faraday's equation and Ampère's equation. (1) describes a non-conservative field in terms of the density momentum, similar to how the time-varying electric and magnetic fields interact in Maxwell's equations. In electromagnetism, the curl of the electrodynamic field relates to the time-varying magnetic field, indicating that a changing source leads to dynamic electromagnetic fields. In this analogy, the density momentum vector $\mathbf{D}$ can be thought of as playing a role similar to the electric field, where the changing density momentum corresponds to the evolution of the field due to dynamic sources. The curl equation of (31) serve as an analogy to the Faraday equation of the electric field around a conductor. This is given as [26]

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \tag{55}$$

Similarly, by examining the density vector around a linear source, we derived the relation in (26). This is analogous to the Ampère Law, which describes the relationship between the current density and the magnetic field, provided as [26]

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}. \tag{56}$$

These equations demonstrates the similarities of the fluid dynamic from a linear source, which we have analyzed in Section 4.2, and the electromagnetic field around a linear conductor.

6 Effect of Motion of Charged Particle in Observed Field

In this section, the analogy of the fluid flow is applied to the relativistic transformation between electric and magnetic fields. In particular, the relationship between the electrostatic, electrodynamic and magnetic field, is clarified, leveraging the previously established framework of the fluid flow.

The key takeaway in Section 4.3 is that motion creates a time-dependent change in the density momentum field relative to the test source, regardless of the moving source. This time-varying density momentum is observed as a non-conservative field by the moving test source, even if the field is stationary and conservative. In this section, the important of the transformation to the reference frame of the test charge shall be explained. In electromagnetism, the electrostatic and electrodynamic fields are different components of the electric field, with the electrodynamic field being created by the relative motion of the charge. Whereas the magnetic field is a property that inherently exist alongside the electrodynamic field. Therefore, when an electric field is said to transform into a magnetic field, the electric field mentioned is actually the electrostatic field, and the statement also signifies that the electrostatic field transforms into an electrodynamic field [27]. The electric field in this context should not be confused with the electrodynamic field, whereby stating that a changing electrodynamic field produces a magnetic field, which is a misconception. The magnetic field or electrodynamic field is produced under some condition by a changing electrostatic field, which shall be explained soon in the following sub-section.

In the rest frame of a static charge distribution, the electric field $\mathbf{E}$ can be described using Coulomb's law. When charges are in relative motion, they produce a relative electrodynamic field, which by Faraday's equation also demonstrate that a magnetic field is produced. However, to produce the electrodynamic field, the motion must be perpendicular to the test source, to satisfy the condition of an electrodynamic field. When it is parallel to the test source, a non-conservative electric field is produced. The field has the same form as that analyzed in Section 4.3, which when applied to the electric field, would give

$$\nabla \cdot \mathbf{E} = -\frac{\partial}{\partial t}\beta, \tag{57}$$

serving as an analogy to (34), the non-conservative equation is distinct from the Faraday's equation, since the equation is not a curl equation and the field density is distinct from the magnetic field vector. To derive Faraday's equation, the multiplication of the equation by the density unit vector is compulsory. The unit vector multiplication would represent the current property which by Ampere's equation has a rotational magnetic property.

This summarizes that an electrodynamic field production depends on the orientation of the motion relative to the test source, otherwise, a magnetic field or electrodynamic field is not necessarily produced. A motion in the same axis relative to a test source does not produce a magnetic field, whereas a motion perpendicular to the axis produces an electrodynamic and magnetic fields.

6.1 The Lorentz Force Equation

In the presence of an electrostatic field, a test charge experiences a force directed towards or away from the source of the charged source. Notably, when a linear source with current possesses a non-zero electrostatic field, the resulting electrostatic, electrodynamic, and magnetic fields are mutually perpendicular. This orthogonal relationship enables us to express the electrostatic field in terms of the electrodynamic and magnetic fields using vector analysis.

The electrostatic field is represented as

$$\mathbf{E}_i = \mathbf{E}_k \times \hat{b}, \tag{58}$$

where $\mathbf{E}_i$ denotes the electrostatic field relative to the test source from the x-direction, $\mathbf{E}_k$ represents the electrodynamic field relative to the z-direction, and $\hat{b}$ is the magnetic unit vector. Substituting $\mathbf{E}_k = \beta \mathbf{v}_k$ into the equation, it yields

$$\mathbf{E}_i = (\beta \mathbf{v}_k \hat{b})_i. \tag{59}$$

Given that $\mathbf{B}$ is related to β as given in (51), the expression is simplified to arrive at

$$\mathbf{E}_i = \mathbf{B}_j \times \mathbf{v}_k, \tag{60}$$

demonstrating that the longitudinal electric field is equivalent to the cross product of the magnetic field and the velocity of the charge relative to the field, which is relatively equivalent to the velocity of the field relative to the charge.

The force experienced by the charge in the longitudinal direction (in the direction of $\hat{\imath}$), is the sum of the electrostatic field and the electrodynamic (or magnetic) field. Consequently, the total force can be expressed as

$$\mathbf{F}_i = q(\mathbf{E}_i + \mathbf{B}_j \times \mathbf{v}_k), \tag{61}$$

which is the renowned Lorentz equation of electromagnetic force [28]. This equation encapsulates the total force experienced by an electric charge in the presence of electromagnetic fields.

The equation is usually demonstrated in two cases, the force between two current carrying conductor and the force of a charge particle in a magnetic field. These two cases shall be analyzed.

For the force of a conductor, it is important to avoid the misconception of the electron being the only charge that reacts to the force and push the whole conductor in the direction of the force. The motion is better studied in term of the atoms of the conductor. When the electrons flow on the conductor, they exit one atom and move into another atom (Fig. 5). The atom can be taken as a system of changing charge, therefore, the equation takes the form

$$\mathbf{F} = \mathbf{B} \times \frac{d}{dt} q \ \mathbf{r}, \tag{62}$$

where $\mathbf{r}$ signifies the direction of the flow and the radius within which an electron is said to leaves or enters the atomic system. The electrostatic term is omitted, because the electrostatic

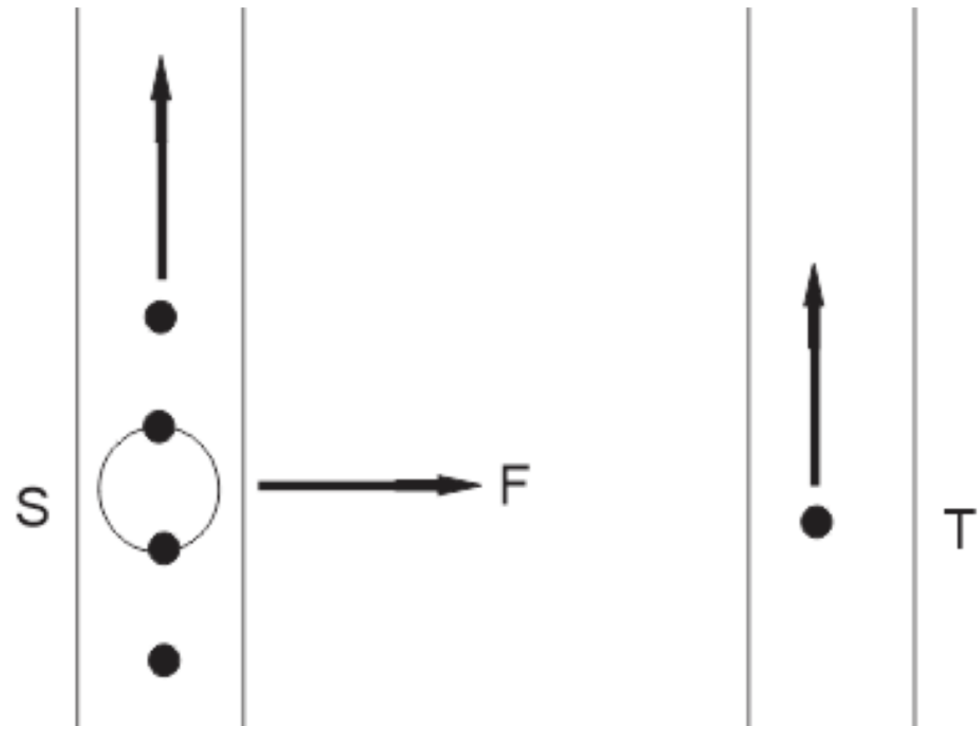

Figure 5: *As an electron leaves the atomic system within the wire S, another electron enters. This is observed as a dynamic positive electric field with direction opposite to the electrons direction. With the current from T producing a magnetic field, the atomic system experience a force in the perpendicular direction. The current is in term of the stationary dynamic positive charge instead of the moving electron.*

field is negligible. An electric force is produced when a magnetic field **B** exists, either produced by a similar current or a magnet. By the cross product, the direction of force depends on the direction of the magnetic field and the charge.

For an electric charge moving through a magnetic field, an electrostatic force is produced when the relative motion of the field is non-zero. That is, the charge experiences a force when it moves relative to the field. It is also important to note that the force, as derived above is a longitudinal force, though being produced by a magnetic field, is still encompassed under the electromagnetic force. Therefore, instead of the electric and magnetic force, the forces are the different form of the same force. The magnetic force is an electrodynamic force which is similar to the electrostatic force, but production-wise, is different.

6.2 The Derivation of the Lorentz Factor

The fluid analogy presented in this paper offers a significant advantage over Maxwell's incompressible fluid analogy. Specifically, it provides a perfect analogical representation of the speed of the field with respect to the test source. In contrast, Maxwell's analogy yields a non-relativistic speed of light or field, failing to account for the relativity of the speed with respect to an observer.

In our analogy, the speed of the field is defined relative to the test source, which is the observer of the force due to the electric field. The effect of the field on the test source is the force of motion, given as the product of the charge of the test source and the electric field of the source relative to it. Therefore, the electric field observed by the test charge, determines it's force of reaction. Therefore, the force of motion of the test charge in the field must be defined in the reference frame of the test charge. The observed electric field is given by

$$\mathbf{E}' = \beta \mathbf{c}', \tag{63}$$

where $\mathbf{c}'$ represents the apparent speed of the electric field.

Consider a test charge moving parallel to a uniform electric field. The electric field observed by the charge would be dynamic and given by

$$\mathbf{E}' = \beta(\mathbf{c} + \mathbf{v}), \tag{64}$$

where $\mathbf{v}$ represents the speed of the charged source relative to the test charge. When the source is moving away, the observed electric field is lesser than the actual field, whereas it is greater when the source is moving toward the test charge, which is represented by the vector $\mathbf{v}$.

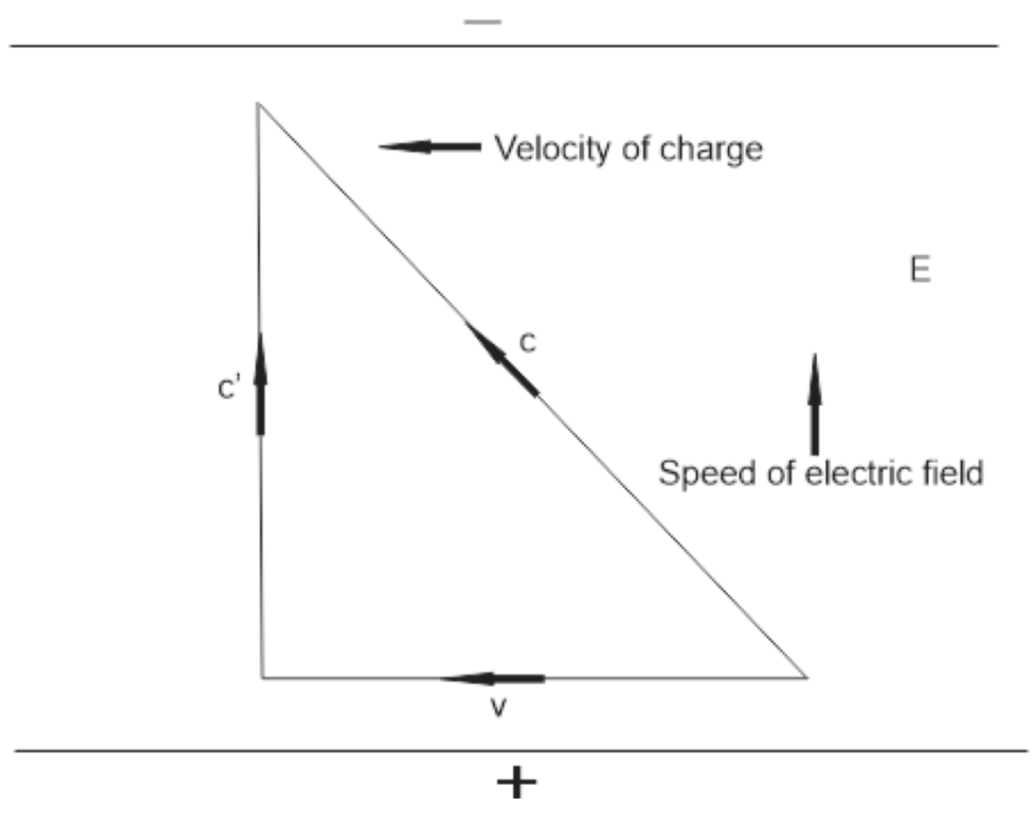

Figure 6: *The velocity of the electric field relative to the test charge (i.e. $\mathbf{v}_{et}$) is given as $\mathbf{c}'$, while the velocity of the test charge relative to the apparatus frame (i.e. $\mathbf{v}_{ta}$ is given as $\mathbf{v}$. The vector sum of these velocities is the velocity of the field relative to the apparatus frame (i.e. $\mathbf{v}_{ea}$), which is the velocity of the field in a relative stationary frame $\mathbf{c}$.*

Now, consider a test charge moving perpendicular to a uniform electric field. In the reference frame of the charge, the electric field observed depends on the its velocity and the perpendicular field velocity, explained in Fig. 6. Using Pythagorean theorem, the relationship is given as

$$\mathbf{c}^2 = \mathbf{c}'^2 + \mathbf{v}^2. \tag{65}$$

This also demonstrates that the field of speed is $\mathbf{c}$ relative to the source reference frame. Substituting the equation in terms of $\mathbf{c}'$ into (63), we get

$$\mathbf{E}' = \beta\sqrt{\mathbf{c}^2 - \mathbf{v}^2}. \tag{66}$$

Simplifying this equation in terms of $\mathbf{E}$, we have

$$\mathbf{E}' = \mathbf{E}\sqrt{1 - \frac{v^2}{c^2}}. \tag{67}$$

The term $\sqrt{1 - \frac{v^2}{c^2}}$ is the inverse of the Lorentz factor [29]. The force observed by the test source would be given by

$$\mathbf{F}' = \frac{\mathbf{F}}{\gamma}. \tag{68}$$

By physical laws, it is consistent for the mass to be constant, leaving the apparent force to be observed as the acceleration of the charge. Therefore, the equation can be written as

$$m\mathbf{a}' = \frac{q\mathbf{E}}{\gamma}, \tag{69}$$

where m represents the mass of the charge and $\mathbf{a}$ represents the acceleration due to the electric field. In order to derive an apparent mass, the equation can be restructured to give

$$\frac{q\mathbf{E}}{\mathbf{a}'} = m\gamma. \tag{70}$$

Defining $\frac{q\mathbf{E}}{\mathbf{a}'} = m'$, the equation becomes the common form [30]

$$m' = m\gamma. \tag{71}$$

This model illustrates the apparent electric force a test charge experiences while moving in a perpendicular electric field, as demonstrated by experiments conducted by Thompson and other researchers [31] [32]. However, it is essential to note that the apparent mass in this equation is misleading and does not signify an actual mass change. Instead, it represents the relativistic force experienced by the test charge and depends on the choice of perspective of experiment interpretation.

7 Application to Electromagnetic Wave

The acceleration nature of the point sources current in the assumed fluid system has been explored. In this scenario, the effect in the field when the current velocity changes is considered. Theoretically, this situation can leads to the wave equation, demonstrating that an an accelerating source can produce a wave of transverse density momentum and density vector field, analogous to how electromagnetic waves are produced in electrodynamics. In this section, we shall descriptively use the flow model to analyze the electromagnetic wave propagation and detection by a distant receiver in the form of a linear conductor (antenna).

7.1 The Electromagnetic Wave Equation

For the fluid source current, the transverse density momentum $\mathbf{D}_T$, and the density vector $\boldsymbol{\rho_f}$ both have directions perpendicular to the current and to one another. As proven, this behaviour is similar to the electric field and magnetic field of a linear current carrying conductor. When both sides of the Maxwell-Faraday's equation is cross multiplied by ∇, and simplified after substituting in the Ampère's equation, the wave equation for the electric field $\mathbf{E}$, is arrived at, provided as [33]

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} = c^2 \nabla^2 \mathbf{E}, \tag{72}$$

where $\mathbf{E}$ represents the electrodynamic field in the analogous electrodynamic system, and c is the wave speed of the electrodynamic field. This equation describes the propagation of electrodynamic field in vacuum or a medium, akin to the propagation of waves in the fluid, as previously analyzed.

Similarly, by applying the same principles to the magnetic field, we obtain [33]

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} = c^2 \nabla^2 \mathbf{B}, \tag{73}$$

where c is the speed of light in vacuum. This equation models the propagation of magnetic waves in the same way the fluid density vector propagates in the analogy.

The wave equations derived above for $\mathbf{E}$ and $\mathbf{B}$ demonstrate how accelerating charge behavior may lead to wave-like propagation of the electromagnetic fields. In the fluid analogy, it was concluded that the changing density momentum and density vector in the fluid may also propagates as a wave. However, it is important to note that the propagation speed in both fluid flow and electromagnetism is governed by the flow of the fluid or field, respectively. Secondly, it is important to note that the equation only suggests that it is possible to produce a wave and does not necessarily mean a wave, or specifically, a sinusoidal wave is produced.

In conclusion, by treating the density momentum and density vector in our assumed fluid flow analogously to the electric and magnetic fields in electrodynamics, a theoretical framework that parallels the behavior of electromagnetic waves in materials with the wave-like behavior in a fluid flow is established.

7.2 Model Analysis of Electromagnetic Induction

Figure 7: *When the current in S creates a transverse electric field (electrodynamic), the field induces a force on T, thereby creating a current. The induced current depends on the distance, which by the model depends on the field density β.*

Electromagnetic induction is a fundamental concept in electromagnetism, where a changing magnetic field induces an electric field in a conductor. This phenomenon is crucial in many applications, including generators, motors, and transformers. In this section, the model of electromagnetic induction by a current-carrying conductor shall be explored.

Consider two linear conductors, S and T, placed a short distance apart (Fig. 7). Conductor S serves as the transmitter or source of the electrodynamic field due to the current, while the electrons within conductor T react to the field by the force

$$\mathbf{F} = q\mathbf{E}_d, \tag{74}$$

where $\mathbf{E}_d$ is the electrodynamic field that reaches conductor T. Since the field is in the same direction as the current, the force of motion by the electrons within conductor T is upwards (Fig. 8).

When the current in conductor S begins to flow upwards, the electrons within conductor T start moving upward, creating a potential difference within the molecules of the conductor. This potential difference is opposed by the attraction of the electrons by the molecules, resulting in an electromotive force (EMF) that acts opposite to the conventional current, as demonstrated by Faraday's law. The conventional current is also opposite to the electron flow. The current flows until the opposing EMF is equal to the induced potential difference, at which point it is broken off. At this voltage, the conductor remains polarized as long as the current remains steady.

If the current in conductor S is suddenly switched off, there is no induced voltage, and the EMF neutralizes the potential difference as the electrons return to their normal positions. This is observed in the electromagnetic induction experiment as the deflection of the galvanometer in the opposite direction.

Similarly, if the current is increased or decreased, the potential difference increases or decreases, respectively, and a small current is observed until the EMF is equal again, opposing any further current. This is the model demonstration of the electromagnetic induction by a current-carrying conductor.

7.3 Electromagnetic Wave of a Current Oscillation

To create a continuous induced current, the current in conductor S must constantly change. This explains the alternating current (AC) methods of generating electricity. When the current from conductor S is reversed, the induced current is reversed as well. Although the induced current is small due to the opposing EMF, the current still exists. Practically, the current is usually amplified.

As discussed in the previous section, a current is induced in a linear conductor when there is a change in the electrodynamic field or magnetic field. A steady current would produce constant electrodynamic and magnetic fields. Similarly, a steady accelerating charge in the current would produce a raising electrodynamic field relative to space and time.

When the current initially has a constant velocity and suddenly reverses its direction, still possessing the same speed, the electrodynamic and magnetic fields both reverse correspondingly. For a periodic reversal of the current, a near-perfect sinusoidal graph is obtained (Fig. 9).

This demonstrates the propagation of the electric and magnetic fields as a wave due to the oscillation of the current. The induced current depends on the strength of the electrodynamic field produced and the proximity of the induced conductor.

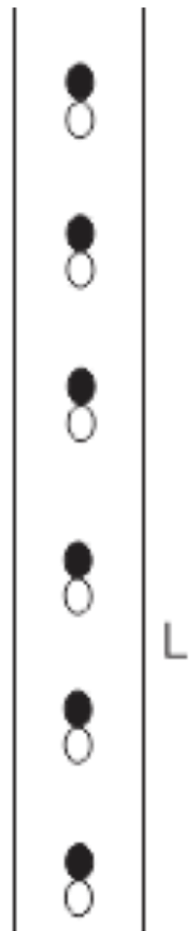

Figure 8: *When the electrons are displaced by the induced current, the attraction by the positive nucleus oppose further motion of the charge. The current stops when the potential difference is equal to the electromotive force.*

This means that for a strong induced current at the same distance, a large current is applied. Also, when the two conductors are close, such that the distance between them is less than the length of the transmitter, the electrodynamic and magnetic fields fall off by the inverse of the distance from the source. This is the near-field.

For the near-field, the complexity of the effect of the finite length can be reduced if the distance is far lesser than the length. However, when the distance is larger than the length of the radius, complexity occurs due to the finite length of the conductor. Whereas if the distance is large such that the source conductor could be assumed as a point source, the electrodynamic field and magnetic field fall off by the inverse square rule.

The model analyzed in this paper demonstrates that the effect of electromagnetic induction is the same effect as the electromagnetic radiation. It is also essential to clarify that the acceleration does not merely produce a wave of the electric field that produces a magnetic field and, in turn, produces an electric field again, which continues off as the wave.

Instead, by this model, the electrodynamic and magnetic fields are produced simultaneously by the current and travel outward from the source.

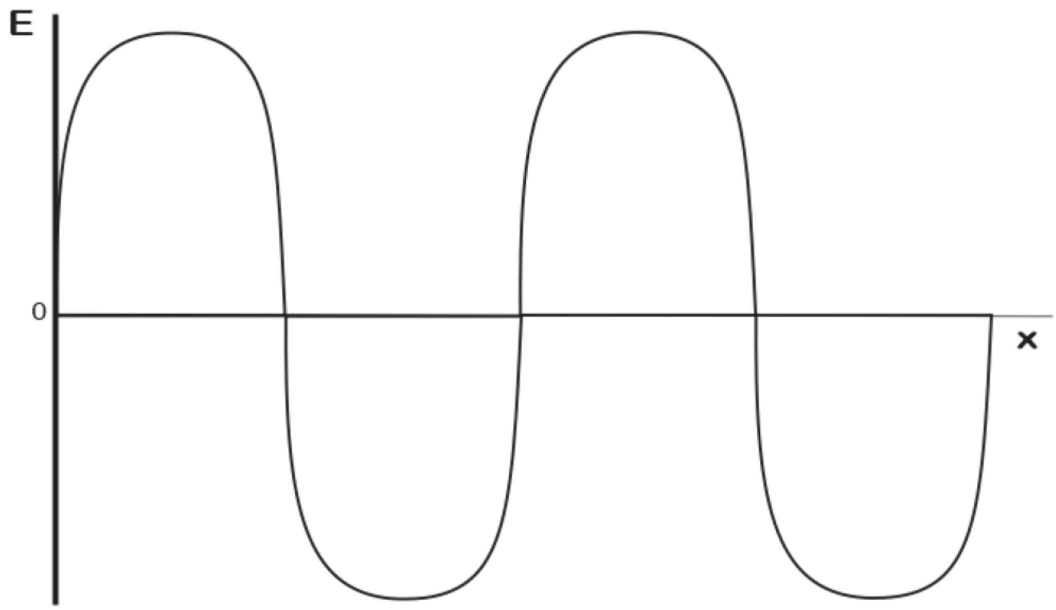

Figure 9: *For an oscillatory electric current, the electrodynamic field produces is oscillatory. The same also applies to the magnetic field.*

8 Discussion and Implications

8.1 Unification of Electric and Magnetic Field

Maxwell's analogy treats the electric field as a divergence field, which emanates from positive charges and converges at negative charges, while the magnetic field is viewed as a curl field [34]. However, in Maxwell's framework, the two fields are not derived from one another to demonstrate their unification. Instead, he described the fields separately, with the only unifying aspect being the relationship within the context of electromagnetic waves. This relationship, however, is more of a connection than a true unification.

In the model presented in this paper, I showed how the electrostatic, electrodynamic, and magnetic fields can be unified within a single theoretical framework. The electric field is analogous to the density momentum field, while the magnetic field corresponds to the density vector field. Both fields are viewed as components of a single, flowing fluid system. The speed of light, denoted c, represents the speed at which the field propagates, implying a finite field effect [35].

By applying this analogy, we can better understand the curl property of the magnetic field. The rotation of the density vector field around a current source explains the magnetic field's curl. This model also provides insight into why magnetic monopoles do not exist. It is crucial to avoid confusing the magnetic field, which is a vector field, with the field density, which is a scalar field. The magnetic field specifically corresponds to the density vector field when the source is in motion—this explains the electrodynamic nature of the magnetic field. The field exists in the direction of the source's motion. While a stationary charge still has an associated field density, this is not the same as the magnetic field, as electrostatic charges do not possess magnetic fields [36].

8.2 Production and Propagation of Electromagnetic Wave

Maxwell's analysis led him to conclude that light is a transverse wave composed of electric and magnetic fields propagating at the speed of light. He also proposed that such waves are produced by accelerating charges. However, the relationship between acceleration and the properties of light has remained unclear, with inconsistencies still unresolved today. The difficulty in reconciling the behavior of accelerating charges with atomic models eventually led to the development of theory, as proposed by Bohr [37].

In this paper, the concept of electromagnetic wave is revisited using the fluid analogy. In this framework, the electric field of a current carrying conductor has two components—the outward diverging electric field and the perpendicular electric field, which is responsible for the wave's propagation and is parallel to the motion of the charge. To produce a complete electromagnetic wave, the source charge must oscillate in a full cycle. A non-oscillating charge would also create an electromagnetic induction, which we have analyzed to be the real effect of electromagnetic radiation, and it radiation frequency depends on the rate of the current change which I did not cover in this paper. However, for a simplified scenario of an oscillatory current, by the principle of energy conservation, oscillations can transfer energy in the form of a wave, which explains why a simple discrete wave frequency is produced by oscillatory current and why continuous wave frequency are produced by irrational charge acceleration. This idea aligns with experimental observations, such as Hertz's work on the production of radio waves through alternating current (AC) oscillations [38] and the brachiation radiation. For an oscillatory current, the frequency of the oscillations is equal to the frequency of the electromagnetic wave.

Likewise, the model of propagation of the electromagnetic wave in this model differs significantly from the ether theory. The ether theory assumes a stationary ether in which transverse waves of it represent the electromagnetic wave of light. The wave is assumed to travel through the ether at the speed of light c, much like a wave propagating through water or a vibrating string. In contrast, the fluid analogy presented here assumes that the fluid itself flows, with two perpendicular density momentum fields and a density vector field, which all propagate outward at the speed of flow. Similarly, in the electromagnetic field, the fields all flow outward through space, including in the outward flow of the wave. The wave is observed as the oscillating perpendicular electric field passing through space, with the speed of the field propagation determining the speed of the wave. This dynamic model contrasts with the older static ether models, where the wave traversed a fixed medium [39]. It also clarify that the speed of light c is not special to the wave, but is the speed of the electromagnetic field. The difference of this model to Maxwell's model is strictly evidential.

8.3 The Nature of the Electromagnetic Field

The finite speed of light plays a crucial role in explaining electromagnetic phenomena. The Lorentz factor, derived from the finite speed of light, demonstrates that the charge's motion through the perpendicular field, affects the electromagnetic field observed relative to the charge. The fluid model also emphasizes the force of the test charge being analyzed in the reference frame of the charge itself and not in the frame of the laboratory. Using the fluid analogy, I have been able to demonstrate common electromagnetism phenomena.

A fluid-like analogy may seem appealing to describe the electromagnetic field, with charges serving as source or concavities in space where the fictitious fluid emanates from or dissipate into. However, this approach faces difficulties in explaining phenomena related to negative sources. Furthermore, the fluid constituent would interact in ways that deviate from observed phenomena, potentially creating new, experimentally observable effects – which have not been reported.

The fluid model in this paper serves solely as an analogy, without introducing new physics. The “fluid” construct, hypothesized earlier, mathematically demonstrates the field’s continuity and dynamic properties. This work proves the dynamic property of the field, contrasting static field theories, by applying fluid dynamics mathematics to model the field.

8.4 Further Analyses

This paper focuses primarily on analyzing the electromagnetic field, it’s forces and the production of electromagnetic waves, providing a conceptual framework based on the fluid analogy. While I have derived key relationships for the electric and magnetic fields, the nature of their interactions and the generation of waves, this work does not extend to a detailed examination of the forces associated with these fields, nor does it delve deeply into the properties of light as an electromagnetic wave, and it transformation in different scenarios. A full model of the electromagnetic force can be derived from the analogy of fluid dynamics, however, it is not included in this paper.

By expanding on these topics, I aim to offer a unified, approach that bridges classical field theory with modern knowledge, providing a more holistic view of the behavior of electromagnetic field and it waves in different contexts.

9 Conclusion

This paper has revisited and expanded upon the classical fluid analogy of electromagnetism, offering a novel approach based on fluid dynamics that diverges from Maxwell’s original incompressible fluid model. By considering a fluid system where momentum, rather than external pressures or potentials, governs the flow dynamics, we have explored a more direct analogy for the propagation and transformation of electromagnetic fields. This approach allows for a better understanding of electromagnetic phenomena, particularly with regard to the propagation of electromagnetic field and their behavior under relative motion.

In this paper, I have introduced the field density, which I have used to define the electrostatic, electrodynamic and magnetic fields, serving as an alternative to the scalar and vector potential. Unlike the potentials which are both needed to account for the three fields (scalar potential for electrostatic, and vector potential for electrodynamic and magnetic). The field density accounts for the three fields altogether, expressing the electrodynamic and magnetic field by the magnetic unit vector $\hat{b}$.

Furthermore, this alternative fluid-based framework has broader implications, potentially informing computational techniques in physics and engineering. The analogy may facilitate the modeling of complex electromagnetic field interactions, providing new insights into wave phenomena and transformations under different reference frames. In particular, the idea of a self-sustained flow, driven purely by momentum, presents a compelling basis for examining electromagnetic wave propagation in ways that challenge traditional assumptions, while preserving intuitive analogies. Beyond it simplification of actual representations, the model still convey the validity of electromagnetism.

In conclusion, by reinterpreting electromagnetic theory through the lens of compressible fluid dynamics, this paper provides a fresh perspective on the dynamics of fields and waves. This approach not only bridges classical fluid mechanics and electromagnetism but also offers a conceptual framework for future investigations into field theory and its relationship to wave phenomena across multiple domains of physics. As the field evolves, continued exploration of such analogies promises to deepen our understanding of fundamental physical interactions and inspire novel methodologies in theoretical and applied physics.

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