An Application of the Hardy Ramanujan Theorem – Proves that Goldbach's Conjecture is "almost always true" by Mathematical Induction

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Abstract. *This paper attempts to answer the riddle of why Ω(m), the sum of the exponents of the prime decomposition of m = (2x-2)(2x-3)(2x-5) …(2x-qb) [where qb is the bth prime and b = π(x), the number of primes not exceeding x], always appears to lie in the region of 2π(x), where π(x) is the number of primes not exceeding x. It turns out that an application of the Hardy Ramanujan Theorem shows Ω(m) is almost always exceeded by 2π(x), which has the logical implication the Goldbach's Conjecture is "almost always true".*

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Introduction

This paper attempts to answer the riddle of why Ω(m), the sum of the exponents of the prime decomposition of a Kamalu product function [Ka], m = (2x-2)(2x-3)(2x-5) …(2x-qb) [where qb is the bth prime, and b = π(x), the number of primes not exceeding x], always appears to lie in the region of 2π(x), where π(x) is the number of primes not exceeding x.

It turns out that an application of the Hardy Ramanujan Theorem shows Ω(m) is almost always exceeded by 2π(x), which has the logical implication that the Goldbach's Conjecture is "almost always true".[HW].

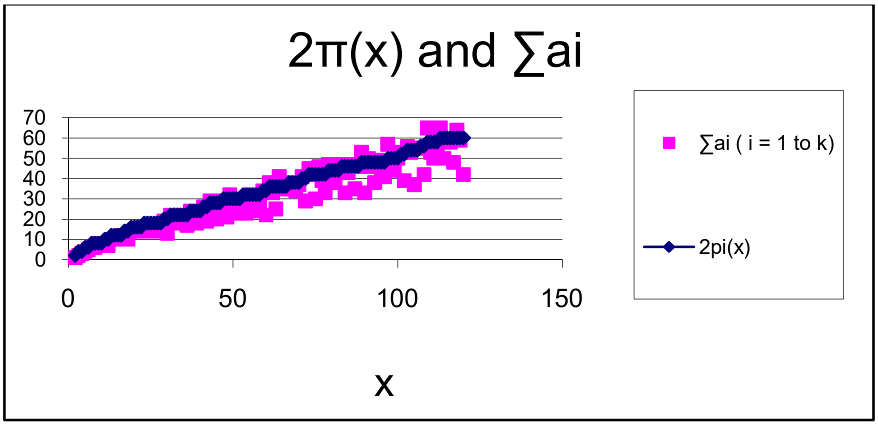

Figure 1 Graph of 2pi(x) compared to Sigma (ai)

The graph in Figure 1 reveals that peculiarly the sum of the exponents Ω(m) = ∑ai *[sigma(ai)]* stays close to 2π(x) *[or 2pi(x) as captioned]* represented by the dark line that rises almost linearly.

Investigation revealed that, when considered in light of an application of the Hardy-Ramanujan Theorem, the function, m, will yield a simple proof that Goldbach's Conjecture (GC) is "almost always true":

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m = (2x-2)(2x-3)(2x-5)...(2x-qb) ....[1], [where qb is the b^{th} prime, and b = pi(x), the number of primes not exceeding x]

The first person to prove that GC is "almost always true" was Estermann in 1929 (published 1938). Here is another proof.

Consider the sum of the exponents in the prime decomposition of m (in accordance with the fundamental theorem of arithmetic),

$m = (p1)^{a1}(p2)^{a2}...(pk)^{ak}$

[where p1, p2,...,pk are the prime factors of m] and call it Ω(m).

Ω(m) = a1 + a2 + ... + ak = ∑ai (i=1 to k)

According to an application of the Hardy-Ramanujan theorem [Ro] (see H.E. Rose, A Course in Number Theory, p.245) we must have for "almost all" values of m (for epsilon > 0) that

$\Omega(m) < (1+ \varepsilon)\log\log(m)$ ....(2)

It so happens that GC is true if

$\Omega(m) < 2\pi(x)$

since one of the factors of m of the form (2x-q) has to then be prime, and so the even number 2x must be the sum of 2 primes, since

$2x = p + q$

where p = 2x-q is prime for some factor of m.

But we note that $m < (2x)^{\pi(x)}$

The reader is left to verify that applying this fact to (2) above and employing mathematical induction, proves that

$\Omega(m) < 2\pi(x)$

since then

$\Omega(m) < (1+ \varepsilon)\log\log(m)$

$< (1+ \varepsilon)\log\log(2x)^{\pi(x)}$

$< 2\pi(x)$

Therefore GC is true "for almost all values of m" and hence GC is true for almost all even numbers 2x.

References

[HW] Hardy, G.H. and Wright, E.M., *An Introduction to the Theory of Numbers,* Oxford University Press, 1979.

[Ka] Kamalu, Chukwunyere, *On Primes of the Form 2x-q (where Q Is a Prime Less Than or Equal to X) and the Product of the Distinct Prime Divisors of an Integer (revised): A Function Approach to Proving the Goldbach Conjecture by Mathematical Induction*." AfricArXiv. February 28. doi:10.31730/osf.io/5u7a8. 2021.

[Ro] Rose, H.E., *A Course in Number Theory,* Oxford University Press, 1994.