Physicsmetrics, another approach to model physics systems. Application on solar panel behaviour. Advantages and complementarities with the physic's approach.

L. Ouzeri[a], F. Hannane[a*], F. Adli[a], M. Aillerie[b,c], J.-P. Charles[b,c], P.Petit[b,c]
[a]*LPTHIRM, University Sâad Dahlab of Blida 1, Soumâa, BP 270, 09000 Blida, Algeria*
[b]*Lorraine University, LMOPS, UR 4423, 57070 Metz, France*
[c]*Centrale Supelec, LMOPS, 57070 Metz, France*
[*] *Corresponding author: fhannanefr@yahoo.fr*

Abstract

The aim of this work is to present two different approaches for modeling a photovoltaic panel (PV) fully illuminated. Generally, the physic's model (White box) takes into account the physic, electronic and energetic behavior of different compound of the system's model, as function of the solar irradiance and operating temperature. The statistician model (Black box) using the Design of Experiment method considers a physical system as a black box with various inputs (factors) and outputs (responses). Each approach has specific advantages, they are complementary.

1.Introduction

Many researchers (whether mathematicians or researchers from other disciplines: physics, chemistry, biology, pharmacy, etc) are unaware of the DoE. Mathematicians, first of all, worry when they see a publication containing words such as photovoltaic, chemical field, etc. They imagine that it is another discipline than mathematics and statistics. As for physicists, chemists, etc, they also worry because when they read the publication (case of the three publications [1,2,3]) they only discover mathematics and statistics throughout the publication, with a minimum of diagrams and information on physics, chemistry, etc. They deduce that the DoE should be taught to all disciplines without distinction in order to reconcile theoretical and experimental research as they are complementary.

Several works have been carried out to model and predict the behavior of a PV panel operating under normal conditions or subject to malfunctions due to shading problems. Generally, the modeling of the operation of a PV system is based on its physical, electronic and energy behavior as a function of illumination and temperature. These models often involve transcendent functions such as exponentials. We introduce a new approach, *Design of Experiments* Method (DoE), which uses statistical tools. We propose the name "*Physicsmetrics*" for this approach when we apply DoE on physics systems. The operation is described using a mathematical model, always in polynomial form, established from experimental measurements.

The physicist's approach consists in representing the PV panel by an equivalent electrical diagram on which the physical laws are applied to obtain a mathematical model linking the current and the voltage delivered. We will present some methods developed by researchers on the determination of the physical parameters constituting the model.

The statistician's approach consists of using measurements made on the panel to establish several models of different parameters to give graphic representations and their interpretations. Advantages and complementarities of each of these two approaches will be given in conclusion in which interesting perspectives for the continuation of this work will be proposed.

2. Photovoltaic effect

*Fig.*1 shows the simple electrical diagram of a solar cell. The current generator I_{ph} (or photoelectronic current) is placed in parallel with a diode carried by the direct current I_d. [4]

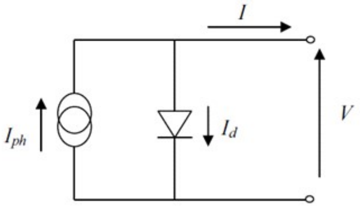


Fig.1. Solar cell diagram.

The experimental variations of the delivered electric current I as a function of the delivered voltage V are given by the characteristic curve I-V shown in *Fig.*2.

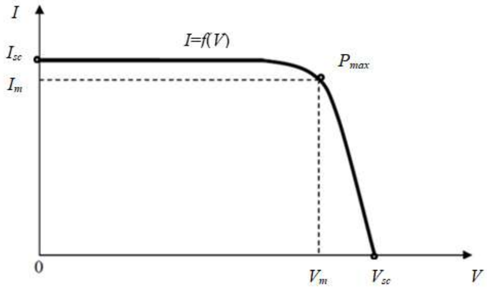


Fig.2. Characteristic curve $I = f(V)$.

Where I_{sc} is the the short-circuit current $(V = 0, I = I_{sc})$, V_{oc} is the open circuit voltage $(I = 0,\ V = V_{oc})$ and P_{max} is the maximum power point $(V = V_m, I = I_m)$.

The mathematical characteristic of a solar cell is given by :

$$I = I_{ph} - I_o \left[\exp\left(\frac{q}{AkT}V\right) - 1\right] \tag{1}$$

Where I_{ph} depends only on the intensity of the incident light energy and therefore on its wavelength, I_0 is the reverse saturation current, A the diode quality factor, q the electronic charge, k the Boltzmann's constant and T the temperature expressed in Kelvin. As shown by *Eq.1*, this model widely used is based on an implicit equation. Moreover, with all these parameters varying with irradiance and temperature this model is also difficult to solve.

By substituting the coordinates of the points I_{sc} and V_{oc} we obtain :

$$V = 0 \qquad I = I_{ph}$$

$$I = 0 \qquad V_{oc} = \frac{AkT}{q}\ln\left(1 + \frac{I_{ph}}{I_o}\right)$$

2.2. Equivalent diagram of a PV

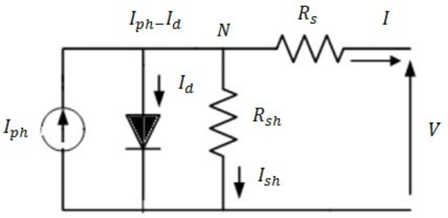


Fig.3. Equivalent diagram of a photovoltaic panel.

*Fig.*3 shows the electrical diagram equivalent to the solar panel. Two resistors have been added: R_s (series resistance), linked to the impedance of the electrodes and the semiconductor and R_{sh} (shunt or parallel resistance), which corresponds to a leakage resistance between the two zones n and p of the junction [4].

2.3. Mathematic model

The laws of electronics make it possible to establish the following model:

$$I = I_{ph} - \frac{V+I\quad_{s}}{R_{sh}} - I_0\left[\exp\frac{q}{AkT}(V+IR_s)-1\right] \qquad (2)$$

The model involves the five parameters: I_{ph}, I_o, A, R_s and R_{sh}, which depend on a set temperature and illumination. If $R_s = 0$ and $R_{sh} = 0$ infinite, we find the ideal equation (1) of the photovoltaic cell.

2.4. The fill factor *FF*

The Fill Factor is defined by the ratio between the maximum power delivered P_{max} and the product of V_{oc} by I_{sc}, $FF = \frac{\mathrm{P}_{max}}{V_{oc}\mathrm{I}_{\mathrm{sc}}}$. This factor measures the quality of the cell. The closer it is to 1, the better the quality of the cell. For a commercial cell, this factor is around 0.70.

3. Main physic's methods for determining the parameters R_s, A, P_{max}, I_o, R_{sh} and I_{ph} of the mathematical model

The methods for determining the characteristic parameters of a photovoltaic panel can be graphical, analytical or numerical. Most are based on approximations.

In this section we will collect some of these methods.

3.1. Determination of R_s [M.Wolf et H.Ranschenbach. 1963] [5]

The series resistor R_s decreases the maximum output power P_{max}, hence the importance of this factor.

We can determine R_s by variant of two levels of illumination I_{ph1} and I_{ph2} (*Fig.*4) and ignoring the effect of R_{sh}. equation (2) will be $I = I_{ph} - I_0\left[\exp\frac{q}{AkT}(V+IR_s)-1\right]$.

For two level of illumination we have $\Delta I_{ph} = I_{ph2} - I_{ph1}$ so:

$$I_2 - I_1 = \Delta I_{ph} - I_0\left[\exp\frac{q}{AkT}(V_2+I_2R_s) - \exp\frac{q}{AkT}(V_1+I_1R_s)\right]$$

since the diode voltage is the same for both illuminations, we can write: $V_2 + I_2R_s = V_1 + I_1R_s$ so $R_s = \frac{\Delta V}{\Delta I_{ph}}$.

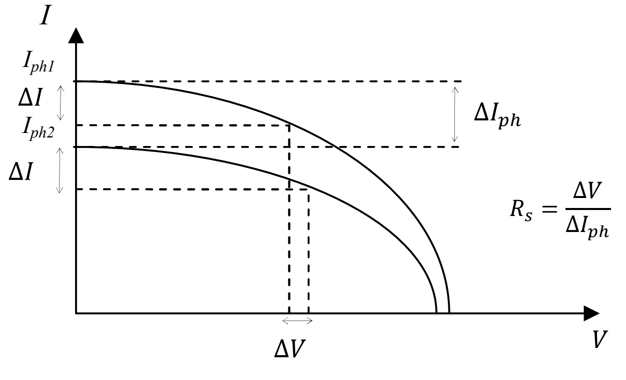


Fig.4. Experimental method of measuring R_s by illumination variation.

Therefore, we can graphically determine the values of ΔV for different values of ΔI in order to deduce the values of R_s.

3.2. Determination of R_s [tangente methode] [4]

This is a quick method which consists in calculating the slope of the tangent of the curve $I(V)$ at the point $V = V_{oc}$.

We concider expression $f(I,V) = I_{ph} - I - \frac{V+R_s I}{R_{sh}} - I_0 \left[\exp\frac{q}{AkT}(V + R_s I) - 1\right] = 0$.

The slope is $p = \left(\frac{dI}{dV}\right)_{V=V_{oc}}$. The derivative of $f(I,V) = 0$ is $df = \frac{df}{dI}\, dI + \frac{df}{dV}\, dV = 0$, where $\frac{dI}{dV} = p = -\frac{\frac{df}{dV}}{\frac{df}{dI}}$. So the slope is $p = \frac{-I_0\ (\frac{q}{AkT})\ \exp(\frac{q}{AkT} V_{oc})}{1+(\frac{q}{AkT})\, R_s\, I_0\, exp(\frac{q}{AkT} V_{oc})}$.

For $V = V_{oc}$, $I = 0$, and because $I_0 \ll I_{ph}$ and $\frac{V_{oc}}{R_{sh}} \ll I_{ph}$ (prouved by numerical values) we have $I_{ph} = I_0\ \exp(q/AkT\, V_{oc})$. The slope becomes $p = -\frac{\frac{q}{AkT} I_{ph}}{1+\frac{q}{AkT} R_s\, I_{ph}}$.

In the condition $\frac{q}{AkT}\, R_s\, I_{ph} \gg 1$ which makes it possible to apply the principle of this method, the slope will be $\frac{-1}{R_s}$ where : $R_s = \frac{-1}{p}$.

this method is advantageous in the study of cells with high concertation and at a constant T, in these cases I_{ph} is large [1].

3.3. Determination of A and R_s [Masatoshi Warachima & Akio Ushirokawa. 1980][5]

Under illumination, neglecting R_{sh} and for the boundary conditions V_{oc} and I_{sc}, the equation (2) gives :

- $I_{sc} = I_{ph} - I_0\ [\exp \frac{q}{AkT}(R_s\, I + V) - 1]$
- $I_{sc} = I_{ph} - I_0\ [\exp \frac{q}{AkT}\ R_s\, I_{sc} - 1]$
- $0 = I_{ph} - I_0\ [\exp \frac{q}{AkT} V_{oc} - 1]$, after eliminating I_{ph} we will get :
- $I_0 = I_{sc} \left[exp\frac{q}{AkT} V_{oc} - \exp\frac{q}{AkT} R_s\, I_{sc}\right]^{-1}$

With the condition $V_{oc} \gg R_s\, I_{sc}$ *i.e* $\frac{q}{AkT}\ V_{oc} \gg\ \exp\ \frac{q}{AkT}(R_s\, I_{sc})$ we get:

$$V = V_{oc} - R_s\, I + \frac{AkT}{q} Log\, \frac{I_{sc}-I}{I_{sc}} \qquad (3)$$

by differentiating V with respect to I: $\frac{dV}{dI}$ give: $R(I) = R_s + \frac{AkT}{q}\frac{1}{I_{sc}-I}$ with $R(I) = -\frac{dV}{dI}$ is the dynamic resistance, its values are obtained from the curve $I = f\ (V)$ from the line $R(I) = f(\frac{1}{I_{sc}-I})$ in the form $Y = \frac{AkT}{q} X$, we deduce R_s the y-intercept and the slope A. (Fig. 5)

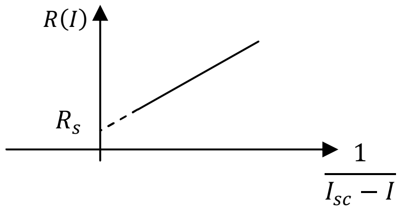


Fig.5. Determination of A *and* R_S.

The advantage of this method is the constant illumination, so no movement of the light source or the cell, nor variation in resistance, except that the condition $\boldsymbol{V_{oc} \gg R_s\, I_{sc}}$ must be respected.

3.4. Determination of P_{max} and of V_m [Masatoshi Warachima & Akio Ushirokawa. 1980] [5]

Given $P_{max} = I_m \times V_m$, we will first determine the value of V_m, we derive P with respect to V and we calculate the zero of the derivative: $\frac{dP}{dV} = 0$.

The equation (3) becomes $\frac{I_{sc}-I}{I_{sc}} = 1 - \frac{I}{I_{sc}} = exp\frac{q}{AkT}(V - V_{oc}\,)\ exp\ \frac{q}{AkT}(R_s\, I)$.

We suppose $\frac{q}{AkT}(R_s\, I) < 1$ then we develop $exp\frac{q}{AkT}(R_s\, I)$ to power series up to 1st order: $exp\frac{q}{AkT}(R_s\, I) \approx 1 + \frac{q}{AkT}(R_s\, I)$ we obtain: $P = V\, I_{sc}\, \frac{1-exp\frac{q}{AkT}(V-V_{oc})}{1+\frac{q}{AkT}(R_s\, I_{sc})\, exp\frac{q}{AkT}(V-V_{oc})}$.

Now we calculate $\frac{dP}{dV}$ by averaging the following approximations:

- $exp\, 2\frac{q}{AkT}\, (V - V_{oc}) \ll exp\frac{q}{AkT}\, (V - V_{oc})$
- $\frac{q}{AkT}(R_s\, I_{sc})\, exp\frac{q}{AkT}\, (V - V_{oc}) < 1$

We obtain $V_m = V_{oc} - \frac{Log[\left(1+\frac{q}{AkT}(R_s\, I_{sc})\right)\left(1+\frac{q}{AkT}V_{oc}\right)]}{\frac{q}{AkT}+(V_{oc}+\frac{AkT}{q})^{-1}}$ where $P_{max} = I_{sc}\, V_m\, \frac{1-exp\frac{q}{AkT}(V_m-V_{oc})}{1+\frac{q}{AkT}(R_s\, I_{sc})\exp(1-\frac{q}{AkT}V_{oc})}$.

The classical method of determining the points (V_m,I_m) consists in finding the biggest value of V.I, which corresponds to finding the slope of the characteristic curve with the *hyperbola* $P = V.I = cte$. This result without approximation depends only on the precision of the measurement (*Fig.*6).

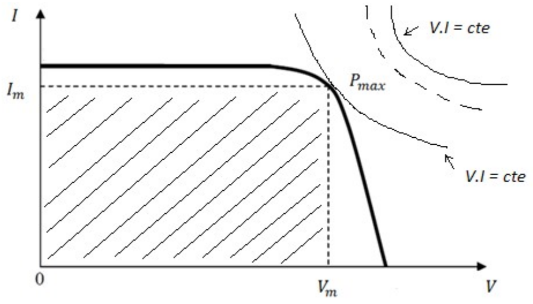


Fig.6. Determination of $P_{max} = (V_m, I_m)$.

3.5. Gaphic determination of A, I_0 and R_s [D.Bonnet] [5]

In dark, $I_{ph} = 0$, when R_{sh} is big we get:

- $R_s\, I \ll V$ because R_s is small
- $\exp(\frac{q}{AkT}V) \gg 1$, so the equation (2) becomes $I = I_0\, .\, \exp(\frac{q}{AkT}\, V)$ so $LogI = LogI_0 + \frac{q}{AkT}V$.

$LogI = f(V)$ is the semi-logarithmic line which gives the slope $\frac{q}{AkT}$ hence the value of A and the ordinate of the origin I_0 :

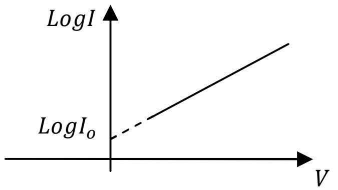


Fig. 7. Determination of A and I_0.

If we cannot neglect $R_s\, I$ in front of V this curve is no longer a straight line, which will only be visible for large values of the intensity I (*Fig.*8): $LogI = LogI_0 + \frac{q}{AkT}(V + R_s\, I)$ where $R_s = \frac{\Delta V}{I}$.

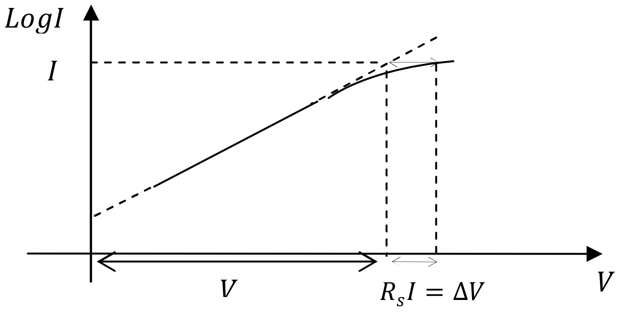


Fig.8. Determination of R_s.

This method neglects the variation of the parameters with the illumination of the solar cell. The logarithmic plot has good linear form if the circuit current is low, $\exp(\frac{q}{AkT} R_s I_{sc}) \sim 1$ and R_s is very low.

3.6. Gaphic determination of R_{sh} [M.Wolf & H.Ranschenbach. 1963] [5]

The value of R_{sh} can be obtained by using the inverse of the characteristic curve in the dark (*Fig.*9):

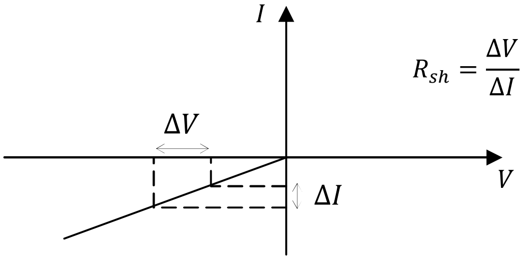


Fig.9. Inverse characteristic in the dark.

Here $I_{ph} = 0$ and in the 3rd quadrant of the current-voltage curve $V < 0$, as soon as $\exp(\frac{q}{AkT} V) \ll 1$ (because $-V \gg \frac{AkT}{q}$) the relation (2) becomes $V = (R_s + R_{sh})I - R_{sh}I_o$. So the slope gives $(R_s + R_{sh})$ where $R_{sh} \gg R_s$. in the dark, the solar cell is considered as a diode. The use of this method requires very effective temperature stabilization. The method always leads to much higher values of R_s.[1]

3.7. Numerical method for determining the parameters I_{ph}, I_o, A, R_s and R_{sh} [J.P. Charles, M. Abdelkrim, Y.H. Muoy & P.Mialhe. 1981] [5]

The method proposed here is a numerical resolution, the parameter A, I_o, R_s, I_{ph} and R_{sh} are determined from data measured relating to V_{oc}, I_{sc} and the corresponding slopes at these points.

- for $I = 0$, $V = V_{oc}$, the equation (2) implies: $0 = I_{ph} - \frac{V_{oc}}{R_{sh}} - I_o\,[exp(\frac{q}{AkT} V_{oc}) - 1]$,
- for $V = 0$, $I = I_{sc}$, the equation (2) implies: $I_{sc} = I_{ph} - \frac{R_s I_{sc}}{R_{sh}} - I_o\,[exp\left(\frac{q}{AkT} R_s I_{sc}\right) - 1]$,

from these two equations we get $\begin{cases} I_{ph} = I_{sc}\ (A_1 - 1) + \frac{V_{oc}}{R_{sh}} \\ I_o = \left[I_{sc}\left(1 - \frac{R_s}{R_{sh}}\right) - \frac{V_{oc}}{R_{sh}}\right]\frac{1}{(A_1 - A_2)} \end{cases}$.

With: $A_1 = exp(\frac{q}{AkT} V_{oc})$ and $A_2 = exp(\frac{q}{AkT} R_s I_{sc})$ and by differentiating the equation (2) we will have $\begin{cases} R_s = R_{so} - \frac{1}{I_o \beta A_1 + \frac{1}{R_{sh}}} \\ R_{sh} = \frac{1}{\frac{1}{R_{sho}} - R_s - I_o \beta A_1} \end{cases}$. R_{so} et R_{sho} are the dynamic resistances, the experimental slopes of the curve $I = f(V)$ in the neighborhood of V_{oc} and the neighborhood of I_{sc}.

$$\begin{cases} \left.\frac{dI}{dV}\right|_{I=0} = -\frac{1}{R_{so}} \\ \left.\frac{dI}{dV}\right|_{V=0} = -\frac{1}{R_{sho}} \end{cases}.$$

For each value of A we get the parameters I_{ph}, I_o, R_s and R_{sh}

The characteristic parameters of the I-V equation are very sensitive for a slight variation of A, mainly in the neighborhood of the point $P_{max}(I_m, V_m)$. So we can deduce A by the equation (2) for $I = I_m$ and $V = V_m$: $I_m = I_{ph} - \frac{V_m + R_s I_m}{R_{sh}} - I_0[\exp\beta(V_m + R_s I_m) - 1]$.

3.8. Determination of I_{ph}, I_0, A, R_s and R_{sh} with Newton-Raphson method [6,7]

It is possible to determine these parameters by iterative methods. The Newton-Raphson method is chosen for the rapid convergence of the response [4]. It is one of the most widely used methods for solving a system of nonlinear equations. The algorithm of this method is based on the use of the Taylor expansion. To solve the equation $f(x) = 0$, starting from an initial value x_0, one seeks a correction Δx such as $0 = f(x + \Delta x)$ by doing a Taylor expansion around $x = x_0$, we find:

$$0 = f(x_0) = f'(x_0).\Delta x + \left(\frac{f^2(x_0).\Delta x^2}{2!}\right) + \left(\frac{f^3(x_0).\Delta x^3}{3!}\right) + \cdots$$

It suffices now to ignore order terms higher than or equal to 2 in Δx to obtain $\Delta x = -\frac{f(x_0)}{f'(x_0)}$.

The correction is the quantity that must be added to cancel the function $f(x_0)$ since we have neglected the order terms higher than or equal to 2 in the Taylor expansion, this correction is not perfect and we put $x_1 = x_0 + \Delta x$ so $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$ where $f'(x_i)$ is the derivative at I, x_i is the actual value and x_{i+1} is the next value.

By extension to the system of equations we get $x_{i+1} = x_i - J^{-1}(i)\, f(x_i)$, J is the Jacobian matrix.

There are three key points in the I-V curve: the short circuit point I_{sc}, the open circuit point V_{oc} and the maximum power point P_{max}. From the characteristic equation (2) we consider the non linear system of 5 unknown variables: I_{ph}, I_0, A, R_s, R_{sh} and 5 equations : $f_1(x)$, $f_2(x)$, $f_3(x)$, $f_4(x)$, $f_5(x)$ such as:

- when $I = 0$, $V = V_{oc}$, $f_1(x) = 0 = I_{ph} - \frac{V_{oc}}{R_{sh}} - I_0\left[\exp\frac{q}{AkT}V_{oc} - 1\right]$
- when $V = 0$, $I = 0$, $f_2(x) = 0 = -I_{sc} + I_{ph} - \frac{I_{sc}R_s}{R_{sh}} - I_0\left[\exp\frac{q}{AkT}(I_{sc}R_s) - 1\right]$
- when $P = P_{max}$, $f_3(x) = 0 = \frac{I_m}{V_m} - \frac{1}{R_{sh}}\left(1 - \frac{I_m}{V_m}R_s\right) - I_0\frac{q}{AkT}\left(1 - \frac{I_m}{V_m}R_s\right)\exp\frac{q}{AkT}(V_m + R_s)$
- when $\left.\frac{dI}{dV}\right|_{I=0}$, $f_4(x) = 0 = -\frac{1}{R_{sh}}\left(1 - \frac{R_s}{R_{so}}\right) - I_0\frac{q}{AkT}\left(1 - \frac{R_s}{R_{so}}\right)\exp\left(\frac{q}{AkT}V_{oc}\right)$
- when $\left.\frac{dI}{dV}\right|_{V=0}$, $f_5(x) = 0 = -\frac{1}{R_{sh}}\left(1 - \frac{R_s}{R_{sho}}\right) - I_0\frac{q}{AkT}\left(1 - \frac{R_s}{R_{sho}}\right)\exp\left(\frac{q}{AkT}I_{sc}R_s\right)$

While $X = (R_{sh}, I_{ph}, I_0, A, R_s, R_{sh})$ we will have the system $f_i(x) = 0,\ i = 1\cdots 5$ and the jacobian :

$$\begin{pmatrix}
1 & 1-\exp\frac{q}{AkT}V_{oc} & \frac{q}{A^2kT}I_0V_{oc}\exp\frac{q}{AkT}V_{oc} & 0 & \frac{V_{oc}}{{R_{sh}}^2} \\
1 & 1-\exp\frac{q}{AkT}(I_{sc}R_s) & \frac{q}{A^2kT}I_0I_{sc}R_s\exp\frac{q}{AkT}(I_{sc}R_s) & -\frac{I_{sc}}{R_{sh}}-\frac{q}{AkT}I_0I_{sc}\exp\frac{q}{AkT}(I_{sc}R_s) & \frac{I_{sc}R_s}{{R_{sh}}^2} \\
0 & -\frac{q}{AkTV_m}(V_m-I_mR_s)-\exp\frac{q}{AkT}(V_m+I_mR_s) & \frac{qI_0}{V_mA^3k^2T^2}(V_m-I_mR_s)[AkT+q(V_m-I_mR_s)]\exp\frac{q}{AkT}(V_m+I_mR_s) & \frac{I_m}{R_{sh}V_m}+\frac{qI_0I_m}{A^2k^2T^2V_m}[AkT-q(V_m-I_mR_s)]\exp\frac{q}{AkT}(V_m+I_mR_s) & \frac{(V_m-I_mR_s)}{{R_{sh}}^2V_m} \\
0 & -\frac{q}{AkTR_{so}}(R_{so}-R_s)\exp\frac{q}{AkT}V_{oc} & \frac{qI_0}{A^3k^2T^2R_{so}}(R_{so}-R_s)(AkT+qV_{oc})\exp\frac{q}{AkT}V_{oc} & \frac{1}{R_{sh}R_{so}}+\frac{qI_0}{AkTR_{so}}\exp\frac{q}{AkT}V_{oc} & \frac{(R_{so}-R_s)}{{R_{sh}}^2R_{so}} \\
0 & -\frac{q}{AkTR_{sho}}(R_{sho}-R_s)\exp\frac{q}{AkT}I_{sc}R_s & \frac{qI_0}{A^3k^2T^2R_{sho}}(R_{sho}-R_s)(AkT+qI_{sc}R_s)\exp\frac{q}{AkT}I_{sc}R_s & -\frac{1}{R_{sh}R_{sho}}+\frac{qI_0}{A^2k^2T^2R_{sho}}(AkT-qI_{sc}(R_{sho}-R_s))\exp\frac{q}{AkT}I_{sc}R_s & -\frac{(R_{sho}-R_{sh})}{{R_{sh}}^2R_{sho}}
\end{pmatrix}$$

The Jacobian can be evaluated with Matlab using “Optimization toolbox”.

We have given above some methods (of the physicist) for determining the parameters of the equation of characteristic IV, we will give in the following sections the method of Design of Experiments (DoE) in order to compare the two approaches in the conclusion.

4. Statistic approach, Design of Experiment Method

Several experiments were carried out, the panel being fully illuminated. The panel is placed in front of a light source. The voltage V and the intensity I of the output current are recorded in order to draw the characteristic curve $I = f(V)$ and determine the maximum power delivered. A mathematical model describing the variations in maximum power as a function of illumination and temperature was obtained. This model is a predictive model. It can inform us about the value of the maximum power delivered at any point of the experimental field prospected.

4.1. Experimental set-up and measurements [1]

Light intensity is measured in terms of distances, with the light source placed respectively 1.10, 2.00 and 2.80 meters from the panel (*SD*). The temperature (*T*) of the panel, is measured in degrees Celsius. The panel consists of four rows of 18 cells each, making 72 cells in total. The first group of cells is formed by rows 1 and 2 arranged in parallel, the second group by rows 3 and 4, also placed in parallel. The two groups are connected together in series (*Fig.*10). *Fig.*11 gives a representation of the shape of the characteristic curves obtained in three different SD and T.

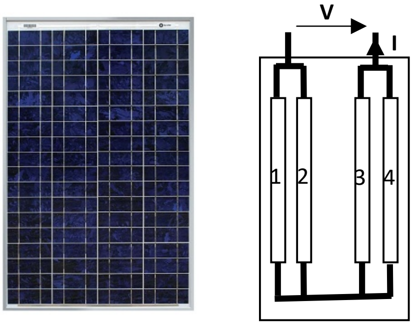


Fig.10. The solar panel.

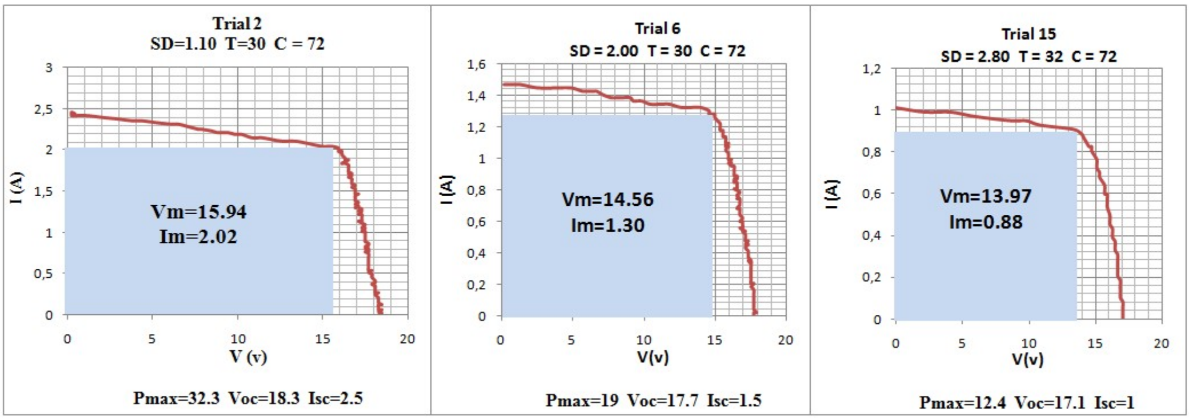


Fig. 11- characteristic curves.

4.2. Basic elements on the Design of Experiments method [8,9]

The system is considered as a black box in which only the inputs and outputs are considered, called factors and responses respectively. The internal constitution of the system is totally ignored.

4.2.1. Mathematical modeling

In the absence of any information about the function that links the response *y* to the factors *x*, we give, *a priori*, the most general formulation $y = f(x_1, x_2, x_3, \cdots, x_k)$ this function is too general and it is customary to take a limited Taylor expansion. Either around $x_0 = 0, y_0 = 0$ we have the MacLaurin series in the second order and changing the notation to:

$$y = f(x_1, x_2) \cong f(0{,}0) + \frac{1}{1!}\left[\frac{\partial f(0)}{\partial x_1}x_1 + \frac{\partial f(0)}{\partial x_2}x_2\right] + \frac{1}{2!}\left[\frac{\partial^2 f(0)}{\partial {x_1}^2}{x_1}^2 + \frac{\partial^2 f(0)}{\partial x_1 \partial x_2}x_1 x_2 + \frac{\partial^2 f(0)}{\partial x_2 \partial x_1}x_2 x_1 + \frac{\partial^2 f(0{,}0)}{\partial {x_2}^2}{x_2}^2\right] + \cdots$$
$$= a_0 + a_1 x_1 + a_2 x_2 + a_{12} x_1 x_2 + a_{11}{x_1}^2 + a_{22}{x_2}^2$$

if the derivatives of the Taylor expansion can be considered as constants, the previous expansion takes the form of a polynomial of big degree: $y = a_0 + \sum_{i=1}^{k} a_i x_i + \sum_{\substack{i,j=1 \\ i<j}}^{k} a_{ij} x_i x_j + \sum_{i=1}^{k} a_{ii} {x_i}^2$

where:

- y is the response measured during the experiment and it is obtained with a given precision.
- x_i represents the level assigned to the factor $i\ (i = 1,2,\cdots,k)$. It is the value of the factor coordinate retained by the experimenter to perform a test. This value is perfectly known.
- a_0, a_i, a_{ij}, a_{ii} are the coefficients of the mathematical model *a priori* adopted. They are not known and must be calculated. They are respectively the value of the response at the center of the field of study, the main effects of the factors, the second-order interactions between factors and the coefficients of the quadratic terms.

The main effect of a factor tells us about the magnitude of the influence of this factor on the response (the greater its absolute value, the more influential it is) and on the direction of variation (the response and the factor vary in the same direction if a_i is positive, in the opposite direction if a_i is negative). Here we have both quantitative and qualitative information on this influence. The experimenter will therefore be able to visually rank the influence of the factors on the response. In particular, he will be able to know the most influential factor. This hierarchy is presented by the DoE software using two diagrams: the histogram of the coefficients of the model and a representation giving the contribution of each of them in terms of percentages. a_{ij} give a measure of the effects of second order interactions between factors. The expression we have chosen is the one adopted in most of the optimization designs called "Response Surface Design". It has been shown that not all interaction can be obtained if we use the old method known as "One factor at a time" and which consists in varying a single factor while keeping the others at constant.

4.2.2. Experimental space and study domain

Each factor is represented on an oriented axis, taking values within its range of variation. The latter is limited by a low level denoted by -1 and a high level denoted by +1. The k axes are orthogonal to each other two by two and constitute a basis of the vector space R^k, of k dimension, called Experimental Space. The combination of the areas of variation of each factor defines the study domain, it is the part of the Experimental Space that the experimenter has chosen to carry out his tests. A study, that is, a well-defined set of experiences, is represented by a series of dots arranged in the study domain. See *Fig.*12.

4.2.3. Coded units

In order to give a generality of presentation to the theory of design of experiments, in order to be free from units and to use dimensionless coordinates, we change the cartesian coordinate system by performing the following two operations: translate the axes to position the new vertex in the center of the study domain, change the units of the axes so that the low and high levels take the values -1 and +1. The transition from the original coordinates to the centered scaled variables (c.s.v), and *vice versa*, is given by $x = \frac{A - A_0}{pas}$ (A_0 being the centered value in current units). For example, ($1.10m$ and $40\,°C$) are the coordinates of the first test carried out. The corresponding reduced centered coordinates become: $x_1 = \frac{1.10 - 1.95}{0.85} = -1$ and $x_2 = \frac{40 - 42.5}{17.5} = -0.1429$.

4.3. Design with 11 trials

Factors are $\begin{cases} x_1 = SD \\ x_2 = T \end{cases}$ and responses are $\begin{cases} y_1 = P_{max} \\ y_2 = V_{oc} \\ y_3 = I_{sc} \end{cases}$ and the experiment space is:

	Level -1	Level 0	Level +1
SD (*m*)	1.10	1.95	2.80
T (*°C*)	25	42.5	60

The study domain is chosen by including all the experimental points carried out, taking for low levels, the minimum values of the two factors (1.10 and 25 °) and for high levels their maximum values (2.80 *m* and 60 *°C*). The distribution of experimental points in the study domain is shown in *fig.*12.

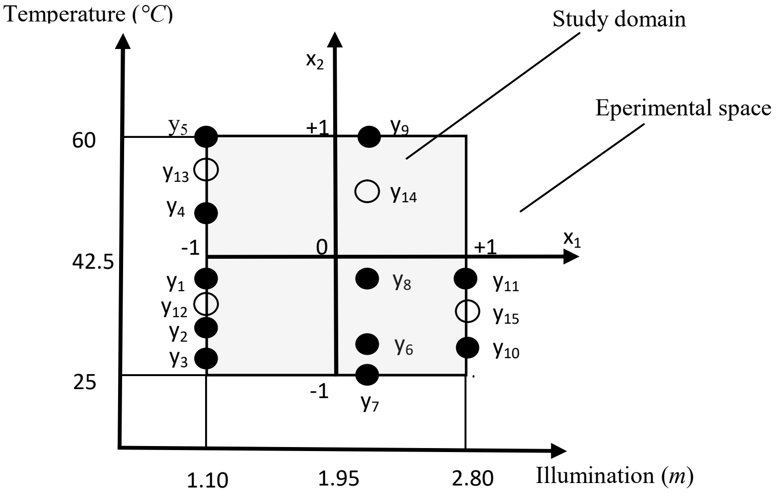


Fig.12. Distribution of the experimental points inside the experimental domain.

4.3.1. Matrix of experiments and test results

The *table*.1 summarizes the data in reel units and in csv.

*Table.*1*. Matrix of experiments and test results*

Essais	*SD* *m*	*Temperature* *°C*	*SD* *csv*	*Temperature* *csv*	V_{oc} *V*	I_{sc} *A*	P_{max} *V.A*
1	1.10	40	-1	-0.1429	18.3	2.5	y_1=32.6
2	-	30	-1	-0.7143	18.3	2.5	y_2=32.3
3	-	26	-1	-0.9429	18.7	2.5	y_3=32.2
4	-	44	-1	-0.0857	18.1	2.5	y_4=31.3
5	-	60	-1	1	17.3	2.5	y_5=31.2
6	2.00	30	0.0588	-0.7143	17.7	1.5	y_6=19.0
7	-	25	0.0588	-1	18.1	1.5	y_7=19.2
8	-	40	0.0588	-0.1429	17.7	1.5	y_8=19.2
9	-	60	0.0588	1	17.1	1.5	y_9=18.4
10	2.80	31	1	-0.6571	17.1	1.0	y_{10}=12.0
11	-	38	1	-0.2571	16.9	1.0	y_{11}=12.5
Level -1	1.10 m	25°C					
Level 0	1.95 m	42.5°C					
Level +1	2.80 m	60°C					

4.3.2. Mathematical model for P_{max}, V_{oc}, and I_{sc}

The expression of the model will include the deviation or residue *e*, a term which combines a model fit error and an experimental error. The model will therefore be $a_0 + a_1x_1 + a_2x_2 + a_{12}x_1x_2 + a_{11}{x_1}^2 + a_{22}{x_2}^2 + e$.

4.3.3. The linear system

If we apply this model to the 11 experimental points, we obtain the following linear system:

$$\begin{cases} y_1 = a_0 - a_1 - 0.1429a_2 + 0.1429a_{12} + a_{11} + 0.0204a_{22} + e_1 \\ \vdots \\ y_i = a_0 + a_1x_{1i} + a_2x_{2i} + a_{12}x_{1i}x_{2i} + a_{11}{x_{1i}}^2 + a_{22}{x_{2i}}^2 + e_i \\ \vdots \\ y_{11} = a_0 + a_1 - 0.2571a_2 - 0.2571a_{12} + a_{11} + 0.0661a_{22} + e_{11} \end{cases} \quad (4)$$

It is a linear system with $n = 11$ equations and $n + p = 17$ unknowns ($p = 6$ coefficients $a_0, a_1, a_2, a_{12}, a_{11}, a_{22}$ and 11 residus $e_1, e_2, \cdots, e_{11}$. We have more unknowns than equations. We can't solve such a system, we are missing 6 equations that will be obtained by formulating an additional hypothesis (least square criteria) and consisting in choosing the vector *a* so that the errors *e* are as small as possible. Therefore the sum of the squares of the residuals be minimum: $\sum_{i=1}^{i=11} e_i^2 = e'e = min$. We give the system (4) in matrix form:

$$Xa + e = y \quad (5)$$

With $y = (32.6 \quad 32.3 \quad 32.2 \quad 31.3 \quad 31.2 \quad 19.0 \quad 19.2 \quad 19.2 \quad 18.4 \quad 12.0 \quad 12.5)'$

The matrix effects *X* and the vector *y* which bring together the 11 measurements carried out are known. The unknown are the vector *a* and *e*. We have in (5) a single matrix relation with 2 unknowns. For any vector *a* chosen arbitrarily, one and only one vector *e* will correspond and *vice versa*.

4.3.4. Matrix of effects *X*

The matrix effects *X* can be easily obtained. We start by writing a column of numbers 1 corresponding to the coefficient a_0 (column *M*), then the columns x_1 and x_2 centered scaled values, of the illumination and of the temperature in Table 1 (columns 1 and 2). We will add a column, product term by term of columns x_1 and x_2 (column 12) and finally the last two columns by taking the squares of the terms of columns x_1 and x_2 (columns 1^2 and 2^2).

$$X = \begin{array}{cccccc} & 1 & 2 & 12 & 1^2 & 2^2 \\ \hline 1 & -1 & -0.1429 & -0.1429 & 1 & 0.0204 \\ 1 & -1 & -0.7143 & 0.7143 & 1 & 0.5102 \\ 1 & -1 & -0.9429 & 0.9429 & 1 & 0.889 \\ 1 & -1 & -0.0857 & 0.0857 & 1 & 0.0073 \\ 1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 0.0588 & -0.7143 & -0.042 & 0.0035 & 0.5102 \\ 1 & 0.0588 & -1 & -0.0588 & 0.0035 & 1 \\ 1 & 0.0588 & -0.1429 & -0.0084 & 0.0035 & 0.0204 \\ 1 & 0.0588 & 1 & 0.0588 & 0.0035 & 1 \\ 1 & 1 & -0.6571 & -0.6571 & 1 & 0.4318 \\ 1 & 1 & -0.2571 & -0.2571 & 1 & 0.0661 \end{array}$$

It is easier to stay in matrix form. The problem therefore is to finding the vector $\hat{a}$ corresponding to the following system: $\begin{cases} y = Xa + e \\ \dfrac{d(e'e)}{da} = 0 \end{cases}$.

Where $e'e = (y' - a'X')(y - Xa) = y'y - y'Xa - a'X'y + a'X'Xa$ the 2nd and the 3rd terms are scalar so $(y'Xa)' = a'X'y$ then $e'e = y'y - 2aX'y + a'X'Xa$ by taking the derivative of the function *e'e* with respect to the vector a ($y'y$ is constant):

$$\frac{d(e'e)}{da} = \frac{d(y'y)}{da} - 2\frac{d(a'X'y)}{da} + \frac{d(a'X'Xa)}{da} = 0 - 2X'y + 2X'Xa = 0 \text{ so } X'X\,\hat{a} = X'y$$

The matrix $X'X$ is a symmetric matrix called *information matrix.* If its determinant is nonzero, we can find its inverse, hence the important formula $\hat{a} = (X'X)^{-1}X'y$, widely used in experimental design theory and allowing to calculate the model coefficients, knowing the calculation matrix *X* and the vector *y* of the measured responses. The inverse matrix $(X'X)^{-1}$ is called the *matrix of dispersion.* By applying this formula, the calculation software will give:

$$\hat{a} = \begin{pmatrix} 19.6519 & -9.8259 & -0.3047 & 0.3107 & 2.5032 & -0.3069 \end{pmatrix}'.$$

So the models will be :

$$P_{\max} = 19.6519 - 9.8259x_1 - 0.3047x_2 + 0.3107x_1x_2 + 2.5032{x_1}^2 - 0.3069{x_2}^2$$

$$V_{oc} = 17.6463 - 0.6034x_1 - 0.4659x_2 + 0.1686x_1x_2 - 0.1559{x_1}^2 - 0.0916{x_2}^2$$

$$I_{sc} = 1.5434 - 0.75x_1 - 0.2066{x_1}^2$$

4.4. Evaluation of the model quality

The experimental and calculated results are very close. The models are therefore of good quality (*Table.*2):

Table 2- Comparison between measured and calculated values

Trials	Measured values			Calculated values		
	V_{oc}	I_{sc}	P_{max}	*Voc*	I_{sc}	P_{max}
1	18.3	2.5	32.6	18.2	2.5	32.1
2	18.3	2.5	32.3	18.5	2.5	32.3
3	18.7	2.5	32.2	18.6	2.5	32.3
4	18.1	2.5	31.3	18.0	2.5	31.9
5	17.3	2.5	31.2	17.4	2.5	31.0
6	17.7	1.5	19.0	17.9	1.5	19.1
7	18.1	1.5	19.2	18.0	1.5	19.1
8	17.7	1.5	19.2	17.7	1.5	19.1
9	17.1	1.5	18.4	17.1	1.5	18.5
10	17.1	1.0	12.0	17.0	1.0	12.2
11	16.9	1.0	12.5	17.0	1.0	12.3

4.4.1. Fill factor *FF*

Trials	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
FF	0.70	0.69	0.69	0.70	0.71	0.71	0.70	0.71	0.72	0.71	0.72	0.70	0.71	0.72	0.71

We notice that the *FF* is good, it is around 0.70.

4.4.2. Precision on the model coefficients

The measured responses are tainted with an experimental error. This error can be estimated by the standard deviation. The matrix X is not a random quantity and it introduces no error. Only the vector y has random variables. The variations of the elements of this matrix therefore lead to variations (or errors) on the coefficients of the model since there is the relationship $\hat{a} = (X'X)^{-1}X'y$. The coefficients determined by the least squares method are random quantities, they will therefore, in turn, generate other random variables when they appear in a mathematical relationship.

Statisticians have demonstrated the following relationship between the variance of the residuals and the variances of the coefficients $V(\hat{a}) = \sigma_r^2(X'X)^{-1}$, relation in which σ_r^2 is the overall variance of the regression, that is, the variance of the residuals. $V(\hat{a})$ is the matrix of the variances covariances of the coefficients. The variances of the coefficients are laid out on the principal diagonal of $V(\hat{a})$ and the covariances are the non-diagonal elements. For the calculation of the standard deviations, we do not need the items corresponding to the covariances. We will only write the main diagonals then the previous formula becomes $DiagV(\hat{a}) = \sigma_r^2\ Diag(X'X)^{-1}$.

It is the relation of the variances of the coefficients. This relation is very important. Indeed, it shows that there are three components which induce errors on the coefficients. These three components are:

- The error made on the answers because of the term σ_r^2. Indeed this term combines the experimental error and the adjustment deviation. The errors made on the measured responses are therefore transmitted to the coefficients of the mathematical model. This is well known, and it comes as no surprise to us.
- The location of the experimental points. The position of the experimental points in the field of study generates greater or lesser errors on the coefficients of the mathematical model. It is the matrix X that is responsible for this transfer. This second result is much more unexpected and deserves to be underlined because it is little known to many experimenters. This means that an experimenter who conducts experiments with great care and obtains very precise test results may still have poor mathematical models if he has misplaced the experimental points in the experimental field.
- The mathematical model chosen *a priori*. The initial choice of the mathematical model made by the experimenter generates more or less adjustment deviations on the coefficients of the mathematical model. Those responsible for this transfer are σ_r^2 and the matrix *X*. This result also deserves to be underlined because it is little known. This means that one should expect large errors on some coefficients when the mathematical model chosen a priori is too far from the real model.

For our example, the matrix being known, we can calculate the dispersion matrix:

$$(X'X)^{-1} = \begin{bmatrix} 0.5297 & -0.0374 & 0.0777 & 0.0309 & -0.3566 & -0.4083 \\ -0.0374 & 0.2840 & 0.1420 & 0.2555 & 0.1639 & 0.0850 \\ 0.0777 & 0.1420 & 0.3437 & 0.2989 & 0.0775 & -0.0141 \\ 0.0309 & 0.2555 & 0.2989 & 0.6789 & 0.1502 & 0.0413 \\ -0.3566 & 0.1639 & 0.0775 & 0.1502 & 0.5180 & 0.1783 \\ -0.4083 & 0.0850 & -0.0141 & 0.0413 & 0.1783 & 0.6326 \end{bmatrix}$$

We apply the formula (6) by keeping only the diagonal elements of the matrix $(X'X)^{-1}$:

$$\begin{bmatrix} V(\hat{a}_0) & . & . & . & . & . \\ . & V(\hat{a}_1) & . & . & . & . \\ . & . & V(\hat{a}_2) & . & . & . \\ . & . & . & V(\hat{a}_{12}) & . & . \\ . & . & . & . & V(\hat{a}_{11}) & . \\ . & . & . & . & . & V(\hat{a}_{22}) \end{bmatrix} = \sigma_r^2 \begin{bmatrix} 0.5297 & . & . & . & . & . \\ . & 0.2840 & . & . & . & . \\ . & . & 0.3437 & . & . & . \\ . & . & . & 0.6789 & . & . \\ . & . & . & . & 0.5180 & . \\ . & . & . & . & . & 0.6326 \end{bmatrix}$$

By identifying the corresponding elements of the two matrices, we obtain the variances of the coefficients:

$$V(\hat{a}_0) = 0{,}5297\sigma_r^2 \quad V(\hat{a}_2) = 0{,}3437\sigma_r^2 \quad V(\hat{a}_{11}) = 0{,}5180\sigma_r^2$$
$$V(\hat{a}_1) = 0{,}2840\sigma_r^2 \quad V(\hat{a}_{12}) = 0{,}6789\sigma_r^2 \quad V(\hat{a}_{22}) = 0{,}6326\sigma_r^2$$

By taking the square roots of these expressions, we have the standard deviation of each of the coefficients:

$$\sigma(\hat{a}_0) = \pm 0{,}7278\sigma_r \quad \sigma(\hat{a}_2) = \pm 0.5863\sigma_r \quad \sigma(\hat{a}_{11}) = \pm 0.7197\sigma_r$$
$$\sigma(\hat{a}_1) = \pm 0{,}5329\sigma_r \quad \sigma(\hat{a}_{12}) = \pm 0.8240\sigma_r \quad \sigma(\hat{a}_{22}) = \pm 0{,}7954\sigma_r$$

If the error of the residuals is 10% then $\sigma_r = 0.1$ and:

$$\sigma(\hat{a}_0) = \pm 0{,}07278 \quad \sigma(\hat{a}_2) = \pm 0.05863 \quad \sigma(\hat{a}_{11}) = \pm 0.07197$$
$$\sigma(\hat{a}_1) = \pm 0{,}05329 \quad \sigma(\hat{a}_{12}) = \pm 0.08240 \quad \sigma(\hat{a}_{22}) = \pm 0{,}07954$$

The standard deviations of the coefficients can be added to the mathematical model:

$$y = 19.65 - 9.83x_1 - 0.30x_2 + 0.31x_1x_2 + 2.5{x_1}^2 - 0.31{x_2}^2$$

The model's multiple correlation coefficient is $R^2 = 0.9991$. It is close to 1. The model therefore has good forecasting quality.

4.5. Graphical representations, analysis and prediction of the behavior of the PV panel

Analyzes and interpretations of the results were carried out using Hide software [10].

4.5.1. Histograms

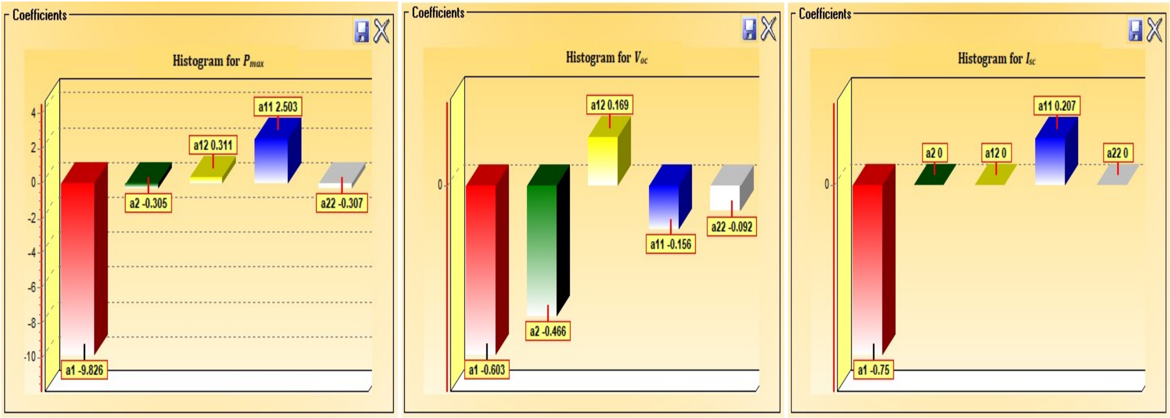


Fig. 13. Histograms.

This representation shows us that the illumination influences more than the temperature on the three parameters P_{max}, V_{oc} and I_{sc}. The interactions between illumination and temperature are all negligible. However, the influence of temperature on V_{oc} must be taken into account.

4.5.2. Sectorial representation

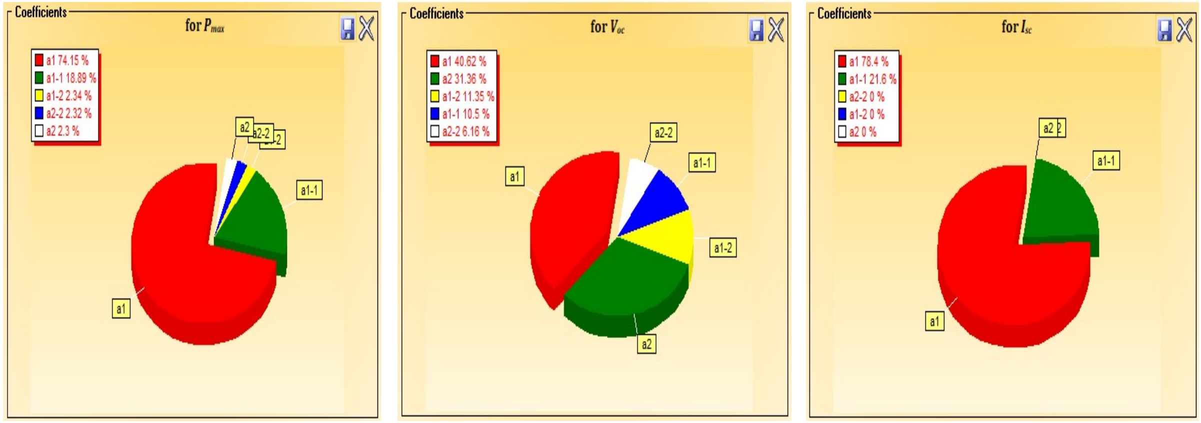


Fig. 14. Sectors.

This representation shows the influence of illumination and temperature on each of the parameters P_{max}, V_{oc} and I_{sc}.

4.5.3. Surface responses

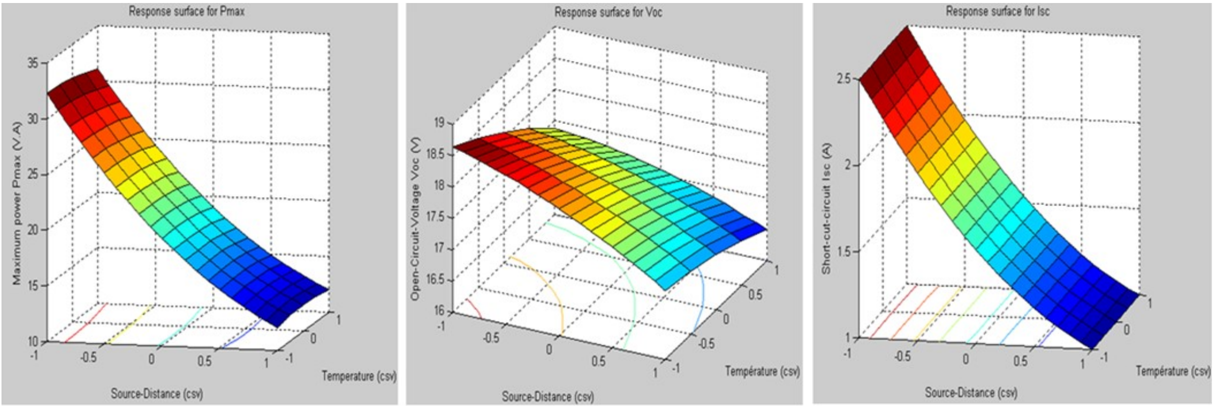


Fig.15. Response surfaces.

The plot of the response surfaces makes it possible to visualize the optimum of each of the parameters P_{max}, V_{oc} and I_{sc}.

4.5.4. Contour lines

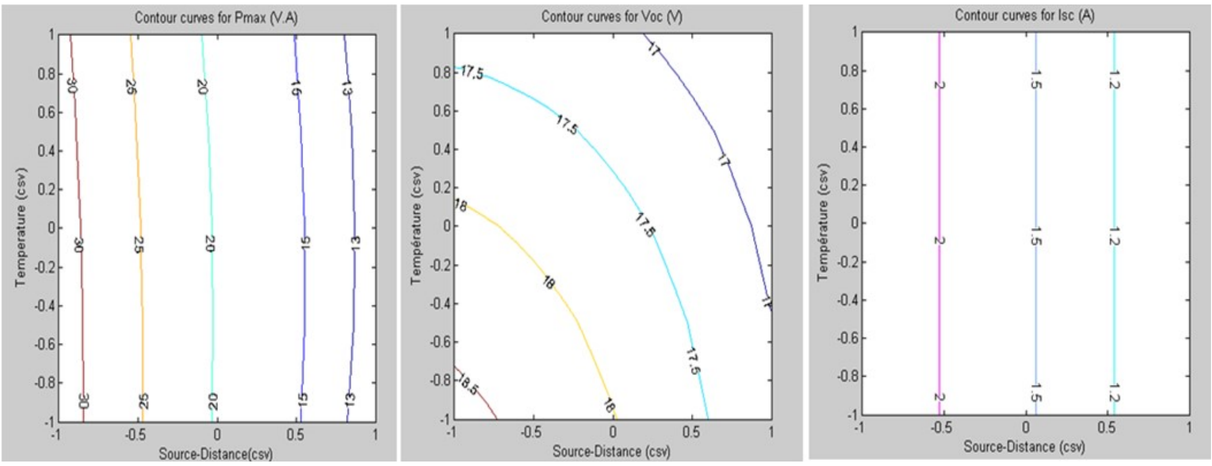
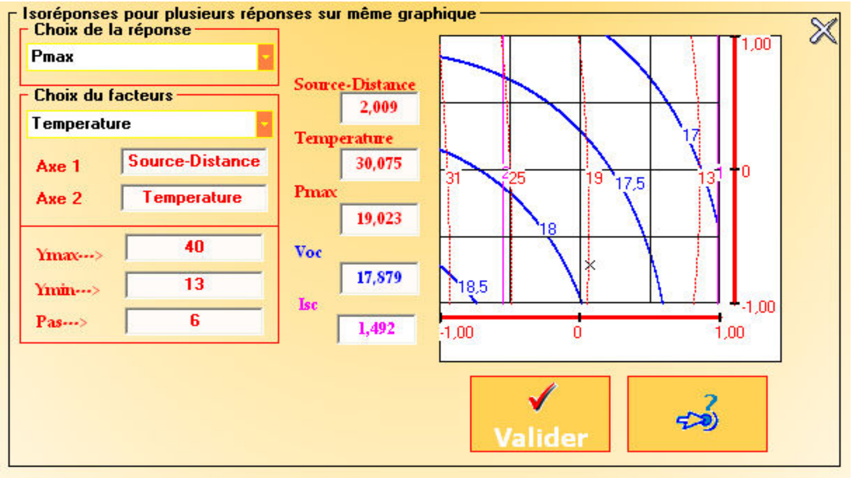


Fig. 16. Contour lines.

These figures show the rate of variation of P_{max}, V_{oc} and I_{sc} as a function of illumination and temperature. We note that P_{max} and I_{sc} vary very little with temperature, although we must take into account the influence of temperature on V_{oc}. The contour lines make it possible to find the optimal conditions in the experimental field.

4.5.5. Iso-responses plotted on the same graph



Isoréponses pour plusieurs réponses sur même graphique

Choix de la réponse: Pmax

Choix du facteurs: Temperature

Axe 1: Source-Distance

Axe 2: Temperature

Ymax--->: 40

Ymin--->: 13

Pas--->: 6

Source-Distance: 2,009

Temperature: 30,075

Pmax: 19,023

Voc: 17,879

Isc: 1,492

Fig. 17. Predict of experimental conditions.

Here the parameters of trial 6 are marked with a cross. By moving the mouse inside the experimental domain we can predict the values of P_{max}, V_{oc} and I_{sc} at any point.

5. Conclusion, advantages, complementarities and perspectives

We have shown in this article that the physicist, for the study of a system is interested in the internal structure of the latter. To study a photovoltaic panel, he begins by representing the system using an equivalent electrical diagram, on which he applies the laws of physics to finally derive a mathematical model. The latter has transcendent functions such as exponentials. The parameters that appear in this model usually have a physical meaning.

The statistician's approach totally ignores the internal structure of the system and he is only interested in the inputs and outputs. The model used is always polynomial, a simple tool. The latter's coefficients on the other hand have no physical significance but they provide information on the properties of the general behavior of the system.

We propose to call the method *Physicsmetrics* referring to the science which uses mathematics, statistics, applied mathematics and algorithms on physic's disciplines.

5.1. Benefits of the physicist's approach

The biggest advantage is that the parameters in the model often have a physical meaning:

- I_{ph} gives the direct measurement of the current delivered by the solar cell.
- A measure the quality of the panel varying between 1 and 3.
- The fill factor FF is a second way to measure panel quality. The closer it is to 1, the greater the power delivered by the panel.
- The physical approach makes it possible to use several methods of determining the parameters of the model: graphical, analytical, iterative and numerical.
- the analytical and adjustment methods on the characteristic curve give comparable values of the parameters of the characteristic I-V.

5.2. Advantages of polynomial representation and the design of experiments method

- It is possible to use other mathematical functions. However, usage shows that polynomials solve most problems and they are the ones that are favored by experimenters.
- The polynomial representation of the model, in addition to its simplicity, allows:
- Apply all the results of matrix algebra due to the linearity of the model with respect to the coefficients.
- To study qualitatively and quantitatively the influence of each factor on the response.
- Establish a hierarchy of this influence (Histograms).
- To qualitatively and quantitatively study second-order interactions.

These four advantages would not have been possible if the expression of the model included transcendent functions such as logarithms and exponentials (disadvantage of the physicist's model).

The main advantage of the design of experiments method is to be able to bring together in a single relationship all the information that can be provided by each factor separately or jointly. This is how with the model, once established, we can (See *Fig.* 13-17):

- Draw the histogram of the coefficients and give a representation in sectoral form of the influence of factors and interactions.
- Calculate the linear correlation coefficients between factors and between factors and response.
- Plot the variations in the response as a function of one parameter while holding the others at constant values.
- Draw the response surfaces and level lines.
- Calculate the value of the response at a point in the study area and plot the prediction error function. The latter makes it possible to control the quality of the prediction of the design of experiments.
- Perform an optimization, ie find the levels of the factors giving the optimal value of the response, before the possible performance of a confirmatory test.
- Incorporate the model into a data acquisition program. One can imagine adapting an acquisition system (lighting and temperature) to the panel. The maximum power delivered in real time will be compared with that given by the model. Any deviation between the two values is a signal of the existence of accidental shading or a malfunction of the panel.
- The use of reduced centered coordinates makes it possible to free oneself from units and therefore to generalize the design of experiments method.
- Carry out the minimum number of experiences in order to have the maximum amount of information in a hierarchical manner.

And the greatest advantage of the design of experiments method is that it can experimentally confirm theoretical results obtained by physicists because this method studies the system as a whole, ignoring its internal structure.

5.3. Perspectives

The subject continues to generate more research work, the following axes are recommended to continue this effort:

- Study with the experimental designs the other parameters other than P_{max}, V_{oc} and I_{sc}.
- Carry out for all the tests the Newton-Raphson method for the calculation of the five parameters I_{ph}, $I_{0,}$ $A_{,}$ R_s and R_{sh}. Then compare with the Hide software.

- Use the experimental designs for the case of a shaded panel.
- Carry out an experimental design to find the ideal doping of a percentage junction.

Références

[1] F. Hannane, H. Elmossaoui, T.V. Nguyen, P. Petit, M. Aillerie, and J-P. Charles. *Forecasting the PV Panel Operating Conditions Using the Design of Experiments Method.* Energy Procedia, 36:479–487, 2013.

[2] J-P. Charles, F. Hannane, H. Elmossaoui, A. Zegaoui, T. V. Nguyen, P. Petit, and M. Aillerie. *Faulty PV panel identification using the Design of Experiments (DoE) method.* International Journal of Electrical Power and Energy Systems, 57:31–38, 2014.

[3] J-P. Charles, M. Aillerie, P. Petit, F. Hannane, and H. Elmossaoui. *Warning of accidental shadowing of a PV generator in operation analyzed with the DoE method.* Solar Energy, 122, 455–463, 2015.

[4] KHOURI Antonio. *Paramètres caractéristiques d'une photopile : Etude expérimentale. Influence de la température.* Thèse de doctorat, Paris, 1986.

[5] ALLAOUI Ismail. *Caractérisation I-V des cellules solaires: Ses problèmes; ses méthodes.* DEA de physiques des solides. Université des sciences et théchniques du LANGUEDOC. 1982.

[6] Adel El Shahat. *PV cell module modeling and simulation for smart grid applications.* Journal of Theoretical and Applied Information Technology, pp 9-20, 2010.

[7] Gwinyai Dzimano,B.S. *Modeling of photovoltaic systems.* Thèse de doctorat. The Ohio State University. 2008.

[8] Goupy J, Creighton L. *Introduction to design of experiments with jmp.* Third edition 2007; SAS Institute.

[9] Goupy J. *Pratiquer les plans d'expériences.* Dunod, Paris 2005.

[10] Elmossaoui H. *Design of experiments software (Hide).*Magister Thesis. Saad Dahlab University of Blida, Algeria, 2004.