# Hybrid Quantum-Classical Computing: A Fusion of Classical and Quantum Computational Substrates

Ravindra Ramouthar[1,2] and Huseyin Seker[3]

[1] *School of Digital, Technologies and Arts, Staffordshire University, Stoke-on-Trent, Staffordshire, UK*
[2] *Microsoft, Redmond, Washington, 98052, USA*
[3] *Faculty of Computing, Engineering and the Built Environment, Birmingham City University, Birmingham, UK*

## Abstract

Through harnessing quantum mechanical phenomena, quantum computing substrates yields the promise of transcending new frontiers in confronting the world's most intractable computational challenges requiring exponential computational power that exceeds that of today's most powerful super-computers. The choreography of such quantum mechanical phenomena has catalysed accelerated advances in the emergence of a quantum computational substrate paradigm that yields the potential to solve some of humanities most complex problems through quantum speedup: in environment, agriculture, health, energy, climate, materials science, precision medicine, autonomous vehicles in smart cities, renewable energy and problems humanity has not yet even imagined. However, the manifestation of a practical quantum advantage across several of these areas is unlikely through the exclusive application of a quantum computational substrate. Quantum computational substrates will not replace classical computational substrates, instead, both technologies will synergistically complement each other where quantum computational substrate accelerators functions as a specialised co-processor to a classical computational substrate in computing workloads best suited for the quantum computer, in a heterogenous computational substrate ecosystem comprising other accelerators. This paper reviewed novel new frontiers at the fusion of quantum-classical computational substrates, and findings reveal that practical applications and research remain limited. The opportunity exists, in considering a heterogenous computational substrate ecosystem comprising other accelerators, for practical quantum advantage across a broader set of complex problem domains to be realised sooner.

## 1. HARNESSING QUANTUM MECHANICAL PHENOMENA

The world is at the cusp of an inflection point that demands the accelerated development and application of quantum computing towards solving some of our planet's biggest challenges - in environment, agriculture, health, climate, materials science, precision medicine, autonomous vehicles in smart cities, renewable energy and problems humanity has not yet even imagined. Classical computing, which manifested the current digital age, is on the cusp of a more exciting, powerful, and radically different form of computing revolution. The dawn of a new technological revolution in computing - known as quantum computing [1], [85] - [87], [30], [88] operates under completely different scientific principles which promises to provide massive technological leaps into new frontiers that suggests disruptive breakthroughs in artificial intelligence, medicine, chemistry, space capabilities, mixed reality, across every industry sector. In assessing the future era of quantum computing and how it will shape the entire landscape, from security to ecommerce and blockchain, [96] concludes that effective strategies are required to transition classical computational technologies into the quantum era. A survey of existing applications, technological advancements, and contemporary challenges associated with quantum computing that aims to distil the promises and limitations of the current state-of-the-art quantum computing supported by use cases and a definition of future research directions has been conducted by [89]. Those left behind will find it hard to catch up. The dire need to venture beyond traditional research boundaries towards making practical contributions that could enhance our daily lives and the collective potential of humanity, driven by an urgent need to provide computational solutions for the world’s greatest challenges that push humanity forward, is now more essential than ever.

## 2. QUANTUM COMPUTING

Classical computers store information as binary 0 and 1 states and process information through bit manipulation. Quantum computers harness quantum mechanical phenomena of superposition and entanglement that manifest states which scale exponentially with number of qubits. The original proposals on quantum computing by

Feynman [1] proposing the use of quantum computers as a scientific instrument towards a deeper understanding of quantum phenomena has since propelled research towards notable developments of superconducting qubits, ion trap qubits, or spin-qubits [2]. Currently, quantum-state entanglement is the accepted fundamental basis of the operation of most quantum technologies categorised into three broad areas: quantum communication, quantum sensing and quantum computing [3]. Quantum technologies with applications in sensing, timing, imaging, communications, quantum simulation and computing that harness the principles of quantum mechanics, had developed over the latter half of the 20th century, based on the premise that quantum systems could be realised through careful engineering that mitigates the effects of the environment [4], [5], [6]. The fusion of science and technology, whereby new science spawns new instruments, and new instruments spawn new science, has established prominence in the emergence of this area of technology around the world, supported through long-term programmes of science and early technology development funding [4], parallels the second quantum revolution that is unfolding [74] - [77].

To manipulate information, quantum computers perform computation through quantum bits, or qubits. Quantum computing hardware can be manifested through several different physical realizations of qubits: trapped ions, semiconducting, superconducting, neutral atoms, electron spin, light polarization, topological qubits, majoranas, nitrogen-vacancy-centers and graphene [7] qubit technologies [8], [2]. DiVincenzo [9] articulates five fundamental principles for the implementation of quantum computation:

1. A scalable physical system bearing a collection of qubits. A qubit is manifested through the states of an atom, or polarization of a single photon.
2. Initialisation of the qubits to a fiducial state such as $|000\ldots>$
3. Decoherence times that are longer than gate operation times.
4. A universal set of quantum gates.
5. A capability to measure specific qubits.

Problems that hinder the realisation of quantum computers include qubit fragility [8], decoherence which is caused by interactions with the environment during calculation and inaccuracies in the induced unitary transformations which result in an accumulation of errors [10] - [12]. Highlighting the differences in error rates between quantum and classical computing, Bertels et al. [2] assert that qubits and quantum gates demonstrate error rates of ($10^{-3}$) compared to CMOS technology ($10^{-15}$) with fault tolerant computation costing more than 90% of computational activity. Another major problem cited by Bertels et al. [2] is the formulation of a quantum logic that harnesses high-performance accelerators for specialised computational kernels that can be exclusively executed on a quantum device. Fault tolerant quantum error correcting codes overcome these limitations once the hardware reaches a certain noise threshold where the error probability is reduced exponentially [10]. The surface code which is a quantum error correcting code proposed by [13] is regarded as the most promising platform for realising a scalable, fault-tolerant quantum computer [11], [12]. Several methods for fault tolerant quantum error correction are currently being explored [14], [11], [15] - [20]. The AWS Center for Quantum Computing has proposed a theoretical blueprint towards fault-tolerant quantum computing that previews a novel approach to quantum error correction (QEC), presenting a comprehensive architectural analysis for the realisation of a fault-tolerant quantum computer based on cat codes concatenated with outer quantum error-correcting codes, with results revealing that with around 1,000 superconducting circuit components, a fault-tolerant quantum computer able to run circuits which are intractable for classical supercomputers can be built and, hardware bearing 32,000 superconducting circuit components, in turn, could simulate the Hubbard model that is beyond the reach of classical computing [21]. More promising, a quantum computer realised through topological qubits is less susceptible to environmental interactions and therefor requires less error correction. [94] discusses the toric code, which is a topological quantum error correcting code and is an example of a stabilizer code/circuit. Topological quantum computation operates on the basis of non-Abelian braiding of quantum states and is mainly focused on the Majorana fermion, which underlies the application of Majorana fermions to create qubit states [94]. Topological qubits are more stable, allows greater system scalability, have a higher reliability endurance and bear a greater resistance to environmental noise [22]. However, according to Wu et al [23], the Jackiw-Rebbi zero-mode promises to be a new candidate for topological quantum computation which has several advantages over the Majorana fermion: (1) the superconductivity is not required; (2) the generalized fusion rule is inherent; and (3) there is a larger energy gap. Simonite [24] reports in a Wired article that in 2018 researchers published evidence on the observation an elusive particle called a

Majorana fermion, however in January 2021 the author and 21 co-authors reported in a new paper based on more conclusive data from their experiments, that they did not find the elusive particle after all [92]. A recent study has found evidence for a cousin of the Majorana fermion [91]. A key scientific breakthrough that demonstrates the elusive building blocks for a topological quantum bit, or qubit has been recently reported [93].

At a fundamental level, Deutsch and Jozsa [25] describe a problem which a quantum computer solves more efficiently and with certainty in exponentially less time than any classical deterministic and stochastic computer. Deutsch and Jozsa [25] assert that the standard mathematical theory forming the basis of classical computing does not account for quantum mechanical effects and the presence of coherent superposition of states during computational evolution. Whilst a quantum computer is able to harness quantum mechanical effects to offer new modes of computation for many classes of problems in a programmable way, it is limited to computing turing-computable functions [25]. According to Deutsch and Jozsa [25], computational tasks either evaluate functions or solve problems, where, in the case of the former, a unique output is obtained, and in the case of solving problems, the goal is to obtain one of several outputs of a specific property. Classical deterministic Turing computers solve problems through reducing tasks to functional evaluation, contrasted with a stochastic computer which does not need to always evaluate functions because its computational evolution and subsequent output is not a function of a unique input. Deutsch and Jozsa [25] assert that a quantum computer harnesses quantum mechanical interference to perform computations which does not need to evaluate functions to solve problems because the state of its output may be a coherent superposition of states representing different answers, each as a solution to the problem. Because the computational state of a quantum computers output is represented as a coherent superposition of states, it is this phenomenon that allows a quantum computer to solve problems that are intractable through classical computing methods. [26] offers evidence that the quantum model of computation offers more complexity theoretic power than a probabilistic Turing machine presented through a problem of distinguishing between two classes of functions that are provably solved exponentially faster through a quantum computation model than a classical probabilistic model when the function is presented as an oracle drawn equiprobably from the uniform distribution on either class. As a derivative of

Simon’s [26] work, Shor [27] has developed the quantum polynomial-time algorithms for the discrete logarithm and integer factoring problems.

For the large-scale feats of quantum systems engineering towards the realisation of scalable quantum computers that are programmable and powerful, short term engineering goals that validate the direction of the designs is essential. In a quantum supremacy experiment devised by Google in the pursuit of a quantum computer that is programmable and powerful, a 54-qubit processor made up of fast, high-fidelity quantum logic gates performed a target computation in 200 seconds that would otherwise have taken the world’s fastest supercomputer 10,000 years to produce the same output [29], [80].

As part of the quantum supremacy experiment as outlined by [29], random simplified circuits from 12 up to 53 qubits were run where the performance of the quantum computer was evaluated using classical simulations and validated against a theoretical model [29]. Classical simulation became infeasible when random hard circuits using 53 qubits were run [29]. The Schrödinger-Feynman algorithm [29], [80] assumes 1M CPU cores for quantum supremacy circuits depicted as a function of the number of qubits and number of cycles for the Schrödinger-Feynman algorithm. This result is pivotal in forging a new frontier in computing manifesting as the first quantum computation that cannot be emulated through classical computing [29]. The results of the quantum supremacy experiment also represent a fundamental challenge against the extended Church-Turing [30] thesis which asserts that classical computers can efficiently implement any “reasonable” model of computation [29]. Scientists from D-Wave recently demonstrated, using quantum annealing, the simulation of some materials up to three million times faster compared to corresponding classical methods. In partnership with researchers from Google, the speed of simulation in one of D-Wave's quantum annealing processors was measured, and results show performance increases with both simulation size and problem difficulty, reaching a million-fold speedup over what could be achieved with a classical CPU and ultimately demonstrating that a quantum computational performance advantage could be achieved over classical means [31]. In a paper published on 3rd December 2020, a team of scientists claimed to have developed the most powerful quantum computer in the world that is capable of performing at least one task 100 trillion times faster than the world's fastest supercomputers. Scientist from the University of Science and Technology of China in Hefei, reported that their quantum computer, named Jiuzhang, is 10 billion times faster than Google's [32]. Quantum computational advantage that exponentially outpaces classical hardware and algorithmic improvements is a function of the ability to scale to a large number of qubits with high-precision control. In work conducted by Wu et al. [33], a two-dimensional programmable superconducting quantum processor called Zuchongzhi, comprising 66 functional qubits in a tunable coupling architecture was developed. The performance of the system was characterised by random quantum circuit where random quantum circuits sampling for benchmarking was performed, up to a system size of 56 qubits and 20 cycles. Results estimate that the computational cost of the classical simulation of this task is estimated to be 2-3 orders of magnitude higher than results from the 53-qubit Sycamore processor. Wu et al. [33] estimate that Zuchongzhi completes the sampling task in about 1.2 hours which will take the most powerful supercomputer at least 8 years. Wu et al. [33] further claim that these results establish an unambiguous quantum computational advantage that is intractable for classical computation in a reasonable amount of time exposing a new frontier of possibilities in the exploration of novel many-body phenomena and the implementation of complex quantum algorithms.

There is a rapid pace of evolution taking place across the quantum computing landscape, driven by large technology vendor platforms seeking to extend their traditional offerings into the quantum computing race. According to [95], in 2023, IBM will unveil Condor, the first universal quantum computer with over a thousand qubits. The company also plans to introduce Heron, a new kind of modular quantum processor that could pave the way for quantum computers with more than 4,000 qubits by 2025. [34] maps many of the key players in the quantum computing ecosystem and [35] – [38] assesses the present quantum computing landscape. In summary, the principles of quantum mechanics and the quantum mechanical phenomena of superposition and entanglement, are permitting quantum computing to perform computations which are much more efficient than classical AI algorithms, and exponentially faster [39], [40] - [47]. However, in the cloud of the projected hype, a key question remains. What are the impactful applications that can manifest a practical quantum advantage. An analysis by [79] reveals that many of the prospective application areas popularized will not benefit from quantum computing unless the algorithms are significantly improved [79].

## 2.1 A FUSION OF CLASSICAL AND QUANTUM COMPUTATIONAL SUBSTRATES

Quantum computational substrates will not replace classical computing, instead, both technologies will synergistically amplify, augment and complement each other where the quantum computer functions as a specialised co-processor to a classical computational substrate in computing workloads best suited for the quantum computer [48], [2], [49] – [51], [70] – [72], [90]. Practical applications of quantum hardware currently remain constrained by the limited number and quality of qubits. A hybrid quantum and classical approach that implements hybrid algorithms harnessing the superior capabilities and features of both classical and quantum computers, overcome the limitations of qubit connectivity, error correction, noise levels and scalability [52]. Whilst classical computers have a larger addressable memory footprint compared to current quantum computers which bear a limited number of qubits, quantum algorithms exhibit superior performance over classical algorithms for specialised computational tasks [73]. The Argonne National Laboratory [52] refer to classical and quantum stages of hybrid algorithms as classical processing units (CPUs) for classical computers and quantum processing units (QPUs) for quantum computers where small graph partitioning problems are solved by the QPU directly whilst larger problems employ a quantum-classical approach. Employing techniques from high-performance computing and classical numerical methods, decomposition methods break down large problems which cannot be run directly on a quantum computer into smaller pieces that the QPU can manage. Hybrid approaches do have their disadvantages, as the size of the problem increases, decomposition methods compromise quantum speedup. To increase the runtime and enable more advanced computation, enhanced qubit quality, qubit count, connectivity, error correction and quantum algorithms will be needed [52]. Bertels et al. [2] highlight the extension of classical computer architecture as emergent accelerators which are coprocessors linked to the host processor that accelerate the execution of specific computationally intensive kernels offloaded from the central processor and therefore speed up the collective execution in line with Amdahl’s law [53].

On innovating at the substrate level, Mohseni and Neven [54], in their patent application describe a microprocessor substrate that combines a classical and quantum processor formed on the substrate with a coupling component that facilitates data transfer between the classical and quantum processors. [54] articulate the following single-chip implementations that combine classical and quantum computing processors:

1. Classical processors execute simple feed forward problems;
2. NP-hard problems and other complicated tasks are executed by the quantum processor and/or a combination of classical processors and the quantum processor;
3. A configuration of the quantum computing processor to receive output data from the classical processor for quantum computation and where the output data can be used to programme the quantum processor;
4. The output port of the classical processor is connected to the input port of the quantum processor through coupling components;
5. The output of the quantum processor is connected to the input of the classical processor through coupling components;
6. The quantum and classical computing processor each contain a superconducting quantum interference device;
7. The quantum and classical computing processor each contain at least one Josephson junction and an inductor;
8. The coupling components contain a superconducting wire;
9. Multiple quantum logic gates arranged according to Reciprocal Quantum Logic (RCL) are contained within the classical computing processor, and
10. Electronic components formed from a superconducting material composed of either aluminium, niobium or lead alloy are included within the classical and quantum computing processors.

Mohseni and Neven [54] highlight the following advantages of single chip implementations of classical and quantum processors:

1. Overall system energy efficiency gains on a single chip implementation of quantum and classical processors cooled to achieve superconductivity which also results in reduced heat dissipation through the classical processor circuits and coupling components;

2. Energy consumption savings of up to eight orders of magnitude through reduced heat dissipation can be realised;
3. Single chip implementations of quantum and classical processors permit the solution of non-trivial hard optimization and inference problems through different classical and quantum metaheuristic algorithms;
4. A single chip implementation provides improved solution accuracy or a reduced time-to-solution where thermal and quantum annealing can be combined on the same chip; and,
5. Single chip implementations also result in shorter distance between the processors, reduced number of communication channels and control lines which result in increased fidelity of state transfer and signal strength.

Co-inventors Garrison, Fano and Weichenberger [55] have been granted a U.S. Patent No. 10,095,981 for a “multi-state quantum optimization engine” that harnesses both classical and quantum computing techniques to enable solutions to intractable problems. The multi-state quantum optimization engine identifies optimal outcomes through running multiple simulations in parallel and is implemented through computer programs through nested calls to classical or quantum devices in one or more locations with its components interconnected through a digital and/or quantum data communication network. Compute resources in the multi-state optimization engine are offered through quantum annealers, quantum simulators, quantum gate circuits which are combined with classical multi-processor compute capabilities that could comprise supercomputer computational capacity [55]. A fusion of classical and quantum computing enables a new frontier of massive scale simulations to determine the best possible outcomes for optimization tasks spanning machine learning, pattern recognition, information security, bioinformatics, image analysis, protein folding, sampling/Monte Carlo, systems design, precision agriculture and scheduling [55].

Hybrid classical-quantum devices are required for a quantum computer to solve classical problems such as complex optimization problems, factoring, searching, complex system modelling. To solve such classical problems, the hybrid classical-quantum device converts the classical problem into a quantum representation, solves the problem and returns the solution as classical data [56]. Existing theoretical single paradigm models of computation lack methods that combine different models into hybrid composites which match real computational devices [56]. Kendon [56] and Lardinois [57] asserts that current experimental quantum processors manifest as hybrid devices where classical controlling hardware is combined with multiple quantum systems that interact through sequences of operations that are precisely specified. The hybrid classical-quantum systems paradigm offers the advantages of better quantum system efficiency and practical experiments [56]. Most applications of quantum technology are expected to be enabled through a hybrid classical-quantum computation paradigm with the quantum computer performing computation as a co-processor [57]. An example of a hybrid classical-quantum paradigm developed by Amazon Web Services, Inc. is a fully managed hybrid classical-quantum computing cloud service that provides a development environment to explore and design quantum algorithms, test them on simulated quantum computers, and run them on several different quantum computers which include gate-based superconductor computers from Rigetti, quantum annealing superconductor computers from D-Wave, and ion trap computers from IonQ [58].

The Microsoft approach to quantum computing harnesses topological qubits resulting in an integrated, scalable solution that combines classical and quantum computing [48]. Azure Quantum is a cloud-based quantum service that offers pre-built solutions, a quantum software development kit and quantum hardware. According to [48], Azure Quantum is a full-stack, open-source cloud ecosystem that enables the development of hybrid classical-quantum solutions that harness classical computers, quantum hardware from Microsoft partners or Microsoft’s own quantum system built on topological qubits. The case for a topological approach, according to [59], is founded on the basis that topological quantum computing requires fewer topological qubits for implementation compared to other systems which increases scalability that results in an integrated solution where classical and quantum computing are combined. Bombin et al. [60] present a comprehensive framework for universal fault-tolerant logic, introducing explicit, but platform-independent representations of topological logic gates and generate new, overhead-efficient methods for universal quantum computation. A survey of open-source tools/Industry tools to develop quantum software has been curated by [61].

According to [62], the hybrid classical-quantum computing model that derives hybrid algorithms which compute against both quantum and classical hardware is regarded as one of the most practical ways to demonstrate useful quantum computation. Chong [62] asserts that the hybrid quantum-classical model has the following benefits:

1. Quantum compute leverages and augments classical computation.
2. Effective utilisation of the limited number of instructions a quantum processor can execute against for classically challenging problems which can be represented by a small number of qubits.
3. Quantum computation can compute classically intractable problems through a hybrid approach therefore making practical quantum computation useful.

The quantum devices that compute quantum algorithms are considered domain specific accelerators given that quantum algorithms are highly specialised. It is for this reason that realising practical applications will require a fusion of specialised quantum processing and classical computing [62]. Chong [62] emphasises that integrated quantum and classical hardware overcomes the challenges of coupling latency associated with quantum and classical computation. Using known input parameters, pure quantum algorithms can be statically compiled with high levels of optimization. However, current challenges for system designers of the hybrid approach include quantum program input parameters that change during each iteration suggesting the need for a partial compilation strategy that is optimized for unchanging parameters and re-optimized for changing parameters [62]. The optimizations are essential in increasing the precision yield of results and [62] highlights the following methods to improve precision:

1. Physical qubit ensembles.
2. Noise reducing circuits.
3. Lightweight error-mitigation codes.
4. Algorithm-level redundancy.
5. Better gradient descent methods.
6. More robust sampling methods across multiple executions.

Bertels et al. [2] define and implement a quantum computer architecture to manifest a quantum computer as a full-stack quantum accelerator. Bertels et al. [2] present three full stack quantum accelerator architecture examples for an experimental superconducting processor, accelerated genome sequencing and near-term generic optimisation problems that are derived from the building blocks approaches defined in (a) Realistic qubits manifested on the experimental full stack, and (b) perfect qubits realised through a simulated full stack. In an endeavour that seeks to maximise the parallelism of high-performance computing systems in the scientific computing domain, an increased emphasis has been placed on a node design architecture that features specialized co-processors acting as accelerators. Work by [63] defines a quantum accelerator framework and architecture for hybrid high-performance computing systems derived from compute nodes containing quantum processing units where selected workloads are offloaded by specialised kernels with the host operating system managing accelerator resources. Britt et al. [63] node design leaves open the question as to what the distinguished differentiators between quantum acceleration and classical parallelism are.

Bertels et al. [2], highlighting Preskill's [64] insights on NISQ technology make for a strong case that the first practical applications will be realised through a hybrid classical-quantum accelerator implementation where specialised computational kernels are executed by accelerators suited to those workloads, with two additional classes of accelerators added. According to Villalonga et al. [64] – [65], Noisy Intermediate-Scale Quantum (NISQ) devices executing on 50-100 qubits exceed the computational capabilities of classical supercomputers, therefore achieving quantum supremacy. In deriving a minimum hardware requirement for quantum computing to exceed classical computation, [65] have developed benchmarks where NISQ supremacy is compared with a state-of-the-art classical high-performance simulator called qFlex reporting an average performance of 281 Pflop/s on Summit, one of the fastest supercomputers in the world, through comparing Random Circuit Sampling (RCS) implementations on both CPU and GPU based supercomputers. The extended implications of Villalonga et al. [65] work also include:

1. The computational capability of different qubit architectures are compared against the classical compute equivalent required.
2. A multi-qubit benchmark for non-Clifford gates is derived.
3. Classical computational architectures are assessed for their simulation of large quantum many-body quantum systems.
4. A comparison of quantum and classical computation energy consumption requirements are evaluated.

5. Through an evaluation of quantum algorithm performance, optimal quantum hardware and algorithm design are informed.

It is envisaged that quantum computers will revolutionize information processing capabilities across a host of military and multi-industry applications spanning pharmaceuticals discovery, to advanced batteries, to machine learning, to cryptography, however a key missing element in the leap toward universal fault-tolerant quantum systems is meaningful metrics that quantify the usefulness of transformative large quantum computers (Defense Advanced Research Project Agency (DARPA), [66]). In an effort to develop standards against which quantum computing progress is quantified and focus current research toward specific goals, DARPA announced its Quantum Benchmarking program which aims to re-invent key quantum computing metrics that are verifiable and estimate the required quantum and classical resources needed to reach critical performance thresholds. DARPA's quantum benchmarking programme seeks to predict the utility of quantum computers guided by an attempt to solve three hard problems (Defense Advanced Research Project Agency (DARPA), [66]):

1. Reinventing key metrics of multi-dimensional scope that define and calculate the delta between current state of the art and quantum computing capabilities.
2. Making metrics testable by creating analogous "wind tunnels" for quantum computers, which currently don't exist.
3. Estimation of the required quantum and classical resources for a given task.

To assess how well a quantum computer can perform, we need more than just checking how accurately it runs some specific circuits: we need to estimate how accurately it can run any circuit. [97] talks about different ways to build models that capture a quantum computer's performance, such as standard error models and machine learning methods, and point out the main scientific challenges in creating reliable performance models. Other works on quantum benchmarking and assessing requirements to scale to practical quantum advantage include [67] - [69], [78]-[79].

Whilst some novel new frontiers at the fusion of quantum-classical computational substrates have been reviewed in this paper, they remain limited, both in their demonstrated practical quantum advantage and volume, therefore exposing a frontier of opportunities and as yet unimagined possibilities for academia and industry to harness quantum speedup in demonstrating practical quantum advantage through a heterogenous computational substrate ecosystem. The exclusivity of quantum computational substrates will not replace classical computing, instead, both technologies will synergistically amplify, augment and complement each other where quantum computational substrate accelerators functions as a specialised co-processor to a classical computational substrate alongside other heterogenous accelerators in computing workloads best suited for the quantum computer.

## CONCLUSION

Through harnessing quantum mechanical phenomena, quantum computing substrates yields the promise of transcending new frontiers to solve problems that present as classically intractable using classical computational substrates. Quantum speedup is achieved by solving problems that harness a quantum computational substrate, where time as a function of problem size, grows slower than that of a classical computational substrate [79]. Beyond the realisation of quantum supremacy, a major frontier lies ahead, that of the pursuit of impactful applications to problems that can realistically yield a quantum speedup – called practical quantum advantage [79]. Finding such impactful applications is relevant to the international research community and multi-industry domains alike, that requires engagement, collaboration and practical contributions. Whilst some novel new frontiers at the fusion of quantum-classical computational substrates have been reviewed in this paper, they remain highly fragmented and limited, both in their practical application and research.

Quantum computational substrates will not replace classical computational substrates, instead, both technologies are expected to have greater collective yield synergistically complementing each other where quantum computational substrate accelerators functions as a specialised co-processor to a classical computational substrate in computing workloads best suited for the quantum computer, in a heterogenous computational substrate ecosystem comprising other accelerators. This paper reviewed novel new frontiers at the fusion of quantum-classical computational substrates, and findings have revealed that practical applications and research remain limited. The opportunity exists, that in considering a heterogenous computational substrate ecosystem comprising other accelerators, that practical quantum advantage across a broader set of complex problem domains may be realised sooner.

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